Sinusoidal Waves: Amplitude, Period & Frequency

Math 117 — Dr. Barksdale

Fundamental Concepts & Notions

Isu # 07c — COHH 4135

Article 00: Readings. Pleasurable and invigorating *reading assignments* follow. Please partake of the intrinsic joy therein found.

Rdng [pp 173 - 177] - - - - - - - - - - - - - - - - - - Exer[p 180] :: #01-26.

Article 01: Sinusoidal Wave Concepts . Consider the graph of $y = \sin(t)$ or of $y = \cos(t)$ as displayed in *FIG_01A & 01B*, below.

(01)

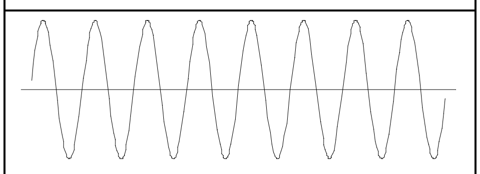

FIG_01A

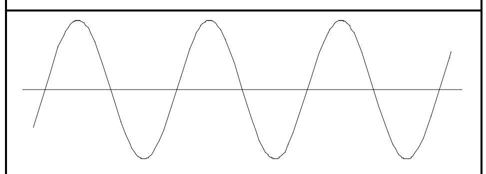

FIG_01B

Such graphs as these are called *sinusoidal wave graphs.* The name, of course, refers to the shape of the *sine function graph.* However, we have already noted that the *sine & cosine graphs* have the same shape. The only difference is the PLACEMENT of the *vertical, y-axis!* Also, note that in the above *wave graphs* the "wave crests" are closer together in *Fig (A)* than those crests in *Fig (B).* Hence, the *cycle length* of the first is SHORTER than that of the second. The (least) *cycle length* is called the *(fundamental) PERIOD* of the function wave. Hence, the *(A)-wave* has a SMALLER PERIOD than the *(B)-wave.* The *period* is the (shortest) *length on the t-axis* which will SPAN ONE FULL CYCLE. The *period* or *cycle length* is also called the *WAVE-LENGTH.* Another feature of *sinusoidal waves* is the *HEIGHT of the CRESTS (from the horizonal axis).* This feature of the wave is called the *AMPLITUDE* of the function wave.

Article 02: Sinusoidal Wave Features. The graphs of $y = \sin(t)$ or of $y = \cos(t)$ are both *PERIODIC with period = 2π* and each has *AMPLITUDE* $= 1$.

(2.1) AMPLITUDE. In order to create *amplitude* $= A$, simply MULTIPLY by A : $y = A \cdot \sin(t)$ and $y = A \cdot \cos(t)$. Now, the *maximum value* of either function will be *A* ; and, the *least value* of either will be $(-A)$. Hence, the *AMPLITUDE* will simply be the *numerical coefficient of the sinusoidal function.*

(2.2) PERIOD (or Wave-Length). Recall that the *periods* of $y = \sin(t)$ or of $y = \cos(t)$ are 2π. Hence, ONE COMPLETE CYCLE of each SPANS a *t-axis length of* 2π. Now, imagine the function $f(t) = \sin(bt)$, for some *real number b* $\in \mathbb{R}$. We ask what *number p* is the *period* of this function $f(t)$? We inspect: $f(t+p) = f(t)$ in order to answer this question. Since, $\sin(t)$ has *period = 2π,* then: $b(t+p) = bt + 2\pi \Rightarrow \sin(b(t+p)) = \sin(bt + 2\pi) \Rightarrow f(t+p) = f(t)$. So, ... solving for p, we find that: $\boxed{p = \dfrac{2\pi}{b}}$. CHECKING: $f(t+p) = \sin b\left(t + \dfrac{2\pi}{b}\right) = \sin(bt + 2\pi) = \sin(bt) = f(t)$.

more / over

(2.3) FREQUENCY. The *FREQUENCY* of a *sinusoidal wave* is simply the *NUMBER of Wave-Lengths (or Period Cycles)* that will *fit into a UNIT length on the t-axis.* Hence, if *TWO* wave cycles will fit into a unit length, then the *frequency is $f = 2$*. Alternatively, if only ONE-THIRD *of a wave length* will fit into a unit length, then the *frequency is* $f = \frac{1}{3}$. Given this DESCRIPTION of *frequency*, it is PLAIN TO SEE that the numerical relation between *period* $= p$ *AND frequency* $= f$ *is given by* $\boxed{f = \dfrac{1}{p} \text{ , or } p = \dfrac{1}{f} \text{ , or } fp = 1}$.

Article 03: Fundamental Observations & Procedural Illustrations.

(3.1) $y = \sin(t)$ and $y = \cos(t)$ have *amplitude* $= 1$ *and period* $= 2\pi$.

(3.2) $y = A \cdot \sin(bt)$ and $y = A \cdot \cos(bt)$ have *amplitude* $= A$ *and period* $= \dfrac{2\pi}{b}$.

(3.3) $y = A \cdot \sin(bt + c)$ and $y = A \cdot \cos(bt + c)$ have *amplitude* $= A$ *and period* $= \dfrac{2\pi}{b}$.

Note that: $\sin(bt + c) = \sin\left(b\left(t + \dfrac{c}{b}\right)\right)$. Now, let $\tau = \left(t + \dfrac{c}{b}\right)$ so that it follows $\sin\left(b\left(t + \dfrac{c}{b}\right)\right) = \sin(b\tau)$. Hence, since $\sin(b\tau)$ has *period* $= \dfrac{2\pi}{b}$, then so does $\sin(bt + c)$.

(3.4) In order to *SKETCH* the *graph of:* $y_t = \sin(bt + c)$, proceed as follows:

(a) First, *sketch the graph of:* $y = \sin(b\tau)$, since this will provide a wave with *period* $= \dfrac{2\pi}{b}$ and such that: $y_\tau = 0$ *at* $\tau = 0$.

(b) Now write: $y_t = \sin(bt + c) = \sin\left(b\left(t + \dfrac{c}{b}\right)\right)$. Then, observe that $t = 0$ *at* $\tau = \dfrac{c}{b}$. So, at $\tau = \dfrac{c}{b}$, *INSERT the vertical y_t−axis.* The graph of $y_t = \sin(bt + c)$ is done!

(3.5) In order to *SKETCH* the *graph of:* $y_t = A \cdot \sin(bt + c)$, *SKETCH as directed* in *Item (3.4) AND provide the wave with an AMPLITUDE* $= A$.

(3.6) In order to *SKETCH* the *graph of:* $y_t = A \cdot \cos(bt + c)$, *SKETCH using PARALLEL procedures to those used for the SINE function.*

Article 04: Concept Examiners. Develop *complete & detailed* solutions for the following morsels of intellectual pleasure!

01. Determine the *AMPLITUDE and PERIOD and FREQUENCY* of each sinusoidal function below:
(a) $5\sin(2\pi t)$ (b) $\frac{5}{8} \cdot \cos(6t)$ (c) $\frac{\pi}{4} \cdot \sin\left(\frac{1}{2}t + \pi\right)$ (d) $\cos\left(8\pi t - \frac{1}{2}\pi\right)$ (e) $\frac{1}{\pi}\cos\left(\frac{t}{\pi}\right)$

02. *SKETCH* each of the sinusoidal functions appearing in *Exercise 01, above.*

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ANS(01): (a) $\boxed{5 \mid 1 \mid 1}$ (b) $\boxed{\frac{5}{8} \mid \frac{\pi}{3} \mid \frac{3}{\pi}}$ (c) $\boxed{\frac{\pi}{4} \mid 4\pi \mid \frac{1}{4\pi}}$ (d) $\boxed{1 \mid \frac{1}{4} \mid 4}$ (e) $\boxed{\frac{1}{\pi} \mid 2\pi^2 \mid \frac{1}{2\pi^2}}$