Nice Polynomials with Four Roots

Abstract: **Nice polynomials are polynomials whose coefficients, roots, and critical points are integers. If the coefficients, roots, and critical points of p(x) are rational numbers, then we call p(x) Q-nice. To begin, we give the relations between the roots and critical points for all polynomials with four roots. We then give the relations between the roots and critical points for all symmetric polynomials with four roots. Using these relations, we derive a formula for all Q-nice symmetric polynomials with four roots. The existence and number of equivalence classes of such polynomials are also discussed. We conclude by giving several examples that illustrate our results.**