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Evolution as Spatial Projection

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Abstract

A theory combining explanations for the nature of three-dimensional systems and evolutionary processes is advanced, set to a geometric simulation, and then discussed in the context of several empirical studies bearing on its validity. The concept "evolution" is first discussed, then related to Baruch de Spinoza's ideas on natural philosophy, including his concept of the conatus. These ideas are then extended by posing the possible existence of a general form of natural systems subsystemization leading to the extended space condition. A geometrical/topological simulation study projecting spatial relations among subsystem structures interpretable through entropy maximization and multidimensional scaling methods is presented, with results providing evidence of the theory's merit. Finally, some past empirical analyses of natural and social systems that also yielded results favorable to the overall idea are summarized, and suggestions for further confirmatory studies are discussed.

Keywords

Evolution, Spatial Extension, Three-Dimensionality, Baruch de Spinoza, Conatus, Multidimensional Scaling, Entropy Maximization

1. Introduction

1.1. Evolution, Some Characteristics

In the present work, a possible internal constraint bearing on the evolution of real-space structures is posed, submitted to a simulation analysis, and shown through empirical example to be relatable to at least some evolved real-world structures. "Evolution" is the term generally applied to discrete systems that begin in an already organized state, and then transform over time into a different, usually or always more organized, state, through an ongoing import and redirecting of energy/information. The naturalist Alfred Russel Wallace (1823-1913) characterized the concept thusly:

> Evolution, as a general principle, implies that all things in the universe, as we see them, have arisen from other things which preceded them by a process of modification, under the action of those all-pervading but mysterious agencies known to us as "natural forces", or, more generally, "the laws of nature". More particularly the term evolution implies that the process is an "unrolling", or "unfolding", derived probably from the way in which leaves and flowers are usually rolled up or crumpled up in the bud and grow into their perfect form by unrolling or unfolding. ([1], pp. 3-4).

It should be noted that Wallace does not specifically restrict the concept here to an association with living things, and in fact we may reasonably think of evolution in other natural or social contexts, for example, in terms of stellar evolution, geocrustal evolution, atmospheric evolution, stream basin evolution, or even human societal evolution. In all these cases, a particular structure comes into being, modifies naturally as is possible according to the flexibility of its particular internal program, and eventually is overcome and destroyed either or both by its own inherent limitations, and/or by events originating from beyond its immediate spatial and/or temporal domain.

And, though this is not usually done, many processes and structures existing at more limited scales of extant and time might also be considered as undergoing a general, if more restricted, evolutionary process: for example, individual cells in organisms, and those same organisms as individual units. In all such instances, regardless of scale, a persisting structure also develops that, after beginning in some lesser form, accepts and utilizes inputs of energy/information, for some limited period of time.

In any case (and whether or not we wish to reserve the exact term "evolution" for certain contexts), there are a multitude of different natural systems of varying scale and duration which are based on a persisting throughput of energy/information. Exactly how this takes place has been dissected in a number of ways, not surprisingly concentrating on apparent characteristics of the particular kind of system under study—stars or biological populations, for example. In the latter context, the mid-nineteenth century work of Darwin and Wallace identified the evolutionary interface between populations and their surroundings we know as "natural selection".

Recently, some researchers (e.g., [2]-[7]) also have concluded that evolutionary processes might be conceived in more generalized terms. Earlier efforts in this direction include the nonequilibrium evolution model promoted by E. O. Wiley and Daniel R. Brooks, and the so-called "constructal theory" proposed by engineer Adrian Bejan [8] [9]. Wiley and Brooks argue that "Evolution may be described as a nonequilibrium process involving the conversion of information from one form to another and the maintenance of old or forging of new reproductive networks. Species participate in nonequilibrium processes because they have properties of closure and because evolution is a historically irreversible phenomenon". ([10], p. 1). Bejan's theory suggests that natural systems evolve in a fashion identifying internal structures that will facilitate the throughflow of the materials that sustain them. Scale is not a consideration, nor is the exact nature of the flows, which might be sustaining systems as different as the organismal vascular system, and human urban commuting networks.

Both of these latter theories, while interesting heuristic devices, may be criticized as falling short of providing exploitable hypotheses. They do, however, avoid one problem that has sometimes plagued cosmology: teleology. The general notion that complexification might be relatable to guided supranatural first causes is anathema to scientific cosmology, which instead features models of probabilistic processes, extending perhaps to teleonomy. In a recent paper, Colin Pittendrigh, an initiator of the teleonomy concept in the 1950s, was described as seeking to "draw a contrast between an external teleology (such as the ends or purposes of a 'creator' or 'designer') and an internal 'teleonomy' (literally 'goal-law'), reflecting the autonomous, goal-directed activities of each and every living being. For Pittendrigh, 'organized' and 'adapted' were virtually synonymous terms, neither of which implied teleology, but both necessarily implied teleonomy". ([11], p. 341). Thus there might be final causes involved after all, but these were conceived either to exist prior to the execution of the evolutionary program, or to somehow emerge during it, as a fundamental property of matter. One understanding related to teleonomy is the anthropic principle, which comes in several varieties depending on the perceived level of inevitability of universal evolution leading to advanced human civilization [12].

A much older, but parallel, idea is fundamental to the philosophy of the rationalist thinker Baruch de Spinoza (1632-1677): the "conatus" concept.

> …Every individual necessarily strives to persevere, and what this involves is an effort to preserve, and even increase, its conatus or power of acting. It is, in effect, a striving to increase its power of striving. …conatus involves "an indefinite duration" and goes right to the heart of a thing's individuation. …a human being, like any finite creature, is a conatus, a determinate, finite parcel of Nature's infinite power that is striving to maintain and increase itself. …Besides explaining what an individual is, conatus also explains why an individual does what it does. If we are talking about human beings, conatus is the motivational force that lies at the root of all of a person's endeavors. …Everything one desires and does, whatever one seeks to achieve or attain or avoid, is—consciously or not—egotistically motivated by the striving to maintain and increase one's power. … ([13], pp. 22-23).

The conatus concept allowed Spinoza to do away with conventional notions of free will, replacing them with the idea that reactive responses to stimuli are only as complete/complex as the individual system is capable of mustering at any particular juncture. Importantly, there seems to be no reason why the concept needs to be restricted to the posed operation of advanced, response-selecting, "finite creatures". The lower organisms also react to stimuli presented by their environment, even if sometimes being little more than automatons in this respect. Indeed, the same is true of individual cells and organs, animal and plant populations, or even physical entities such as whole planets or solar systems. This returns us to our observation that a variety of scales and durations characterize those natural systems that "evolve", yet all of these share the characteristic of persistence for as long as their basic source of energy and materials continues to exist, and sustains a stable throughput process.

Such thoughts leave us to ponder how Spinoza's alleged individual conatuses might be constituted to produce spatial extension itself: otherwise put, can there be a single principle that underlies the physical reality of three-dimensional existence, extending to all of its individual manifestations?

1.2. Three Dimensionality

Spinoza apparently believed so. In part of his *Ethics* [14], he outlines his natural philosophy, resting on the notion that the whole of existence can be characterized through two great "Attributes": Thought, and Spatial Extension, which, accordingly, represent something like rules of order under which everything that is finite exists. In 1986, CHS suggested that perhaps it was the case that all finite natural systems shared some same basic formula of subsystemization: that is, in a manner leading to a balanced input-output of energy/materials that sustained persistence in each individual system, regardless of its scale of existence [15].

From time to time, the question has arisen of why we exist, at the mesoscale at least, within a three-dimensional reality (e.g., [16]-[22]). We are not aware that this question has ever been satisfactorily answered, and will now offer one possible explanation. This is, that extended space is the projection of the interactions among the posed subsystem flows. Otherwise put, perhaps each "conatus-based" system self-organizes (*i.e.*, processes resources/information) through a generalizable internal plan of subsystemization—one in which the flows of materials/information from one to another balance out in the manner of an equilibrial input-output arrangement at any particular instant. This leads to the suggestion that extended space is more than a universal geometry "within which" discrete systems s come and go, but instead a universal characteristic of each and every individual one of those systems, at every, and overlapping, scale of size and time. It is thus posed here to be a condition of existence that each of these entities may maintain internal subsystem relations that project as a three-dimensional state. All other generalizable kinds of interactions between or among natural systems are thereby regarded as epiphenomenal (a good example of such is natural selection itself, which devolves as an interaction among limited resources, population variation and superfecundity). The model does not preclude the possibility of multi-causality at different scales at one time and place, as the epiphenomenal interactions between systems and their environment might eventually lead to system development at multiple scales (for example, cells interacting to sustain organismal units, or the latter combining into a gene pool/population). Further, such epiphenomenal interactions might also usefully absorb system degenerations, as when organismal death reduces tissue to molecules and atoms that individually represent more lowly organized conatuses that are still functioning entities, both unto themselves, and as contributing actors in larger-scale processes (*i.e.*, biogeochemical cycles).

The preceding philosophical discussion suggests that our three-dimensional "real-world" space may be the result of the interactions among some small number of subsystem-level information flows. This notion might be operationalized scientifically: 1) through simulation, if one concedes the possibility that numerical relationships of similarity/influence between discrete structures might be used to identify spatial (geometric) contexts, and/or 2) through empiricism, if direct expressions of the subsystemization process can at least in some instances be predicted to manifest themselves as distinguishable natural patterns. As the supposition here is of a subsystemization structure characterized by input-output relations, a logical starting point for simulation efforts is thus whether certain classes of input-output matrices might consist of values that can directly define a spatial context. To move in that direction, we need first to pause and familiarize ourselves with the analysis technique known as multidimensional scaling.

2. Methods and Results

2.1. Multidimensional Scaling

Multidimensional scaling (MDS) is a very commonly-employed data mining technique in which the relative degrees of similarity/dissimilarity or shared information among the elements of a set of objects are exposed by projecting the net similarities as a configuration of points spread out across a Cartesian, n-dimensional, configuration space. Davison ([23], p. 1) provides the following useful introduction:

> Multidimensional scaling (MDS) is a technique developed in the behavioral and social sciences for studying the structure of objects or people. It has been used to study such things as the social structure of people in an organization, the semantic structure of words, and the logical structure of job tasks for a given organization. …In that MDS is used to describe structure, it is similar to factor analysis and cluster analysis. The assumptions of MDS differ from those of factor analysis and cluster analysis and, consequently, MDS usually leads to a description of the structure which differs in one or more respects from the description provided by the other techniques. …The basic data in MDS are measures of proximity [similarity/dissimilarity] between pairs of objects.

In a typical example, the degrees of similarity among x objects or behaviors are rated by a group (y) of human individuals, leading to the preparation of an $x \times x$ matrix of mean similarities scores. For example, the human subjects may be asked to rate their impressions of the degree of scaled similarity between each pairing of a set of $x = 10$ automobiles. Importantly, the underlying criteria for their ratings may be complex (but are unstated), for example keying on relative sizes, prices or economy. This cryptic criteria structure behind the ratings may be exposed when the latter are transformed into an actual spatial metric, or mapping, involving as many (n) Euclidean dimensions as might be needed (given degrees of freedom considerations) to identify the main influences involved.

In a trivial, but parallel, example, one can imagine a 25×25 matrix of the airline distances among some 25 American cities. Were MDS applied to such a matrix, and a solution in $n = 2$ dimensions called for, a flat-map-like configuration of points corresponding to the cities' actual relative locations would be produced. As in cartographic mappings, however, the reduction of what occurs in three dimensions to two would result in some distortion in the scaled output; that is to say, some of the variation among the real world distances would remain unexplained in the MDS solution (this imperfection of correlation is termed its "stress", and is usually measured as an inverse positive Pearson correlation statistic (r) [*i.e.*, 1.0 − r]). However, an $n = 3$ dimensional MDS solution would provide an unbiased ("zero-stress") mapping of points—a spatial projection—corresponding exactly to the relative actual locations of the cities along the curved surface of the earth.

In practice, the choice of "best n-dimensional projection" is much like choosing the "best" number of factors or clusters in factor or cluster analysis: a trade-off between proportion of variation (in the original data) explained, and degrees of freedom. In MDS, unlike its two cousins, a solution is generated through an iterative algorithm beginning with the arbitrary (usually random) establishment of an n-dimensional array of x coordinate locations. The array is then shuffled and the distances among the elements compared to the original data set; with improvements of fit retained, the elements re-shuffled, and so on, until the output converges to stable solutions that minimize the stress statistic. The end result is a best-fit "mapping" of x coordinate points set in a pre-specified n-dimensional space (with, again, the "n" roughly corresponding to the number of factors obtained in factor analysis).

Since its two pioneering studies appeared in the early 1960s [24] [25], MDS has been in regular use, with hundreds (thousands?) of applications appearing yearly in the academic literature. In the present work, however, an entirely different kind of use for the technique is suggested, one involving a unique kind of hypothesis testing. It is straightforward that in a zero-stress MDS configuration, the projection of the input data into the chosen output dimensionality constitutes a mapping with no "fuzziness" of representation (*i.e.*, the correlation between the original similarities [or dissimilarities] data and the relative positions of the elements of the output configuration is $r = 1.0$); this was the case in the $n = 3$ "25 cities" example provided earlier. Here, however, we are interested, not merely in obtaining an $r = 1.0$ correlation between the input values and the output configuration, but also in identifying stress-free conditions in which each x element is equally weighted on the set of $n = 3$ dimensions: thus, each point being both the same distance to the origin (0, 0, 0), and having the same set of distances to the other points in the configuration. The effect is that, rotationally speaking, all of the points in the configuration are positionally indistinguishable from one another.

We are thus proposing the possible existence of a form of natural subsystemization in which each of the units share information with one another in a manner that directly projects as three-dimensional space. Some manner of energy cascade from one unit to the next is implied (as per Bejan's theory mentioned earlier), but speculation along those lines is getting ahead of ourselves; more centrally, as a point of departure for discussion here, each unit is posed to contribute equally, as expressed by its weighting on the three dimensions, to the overall integrity of a given structure.

This last represents a problem for practical, real world, measurement, since the importance of any given subsystem-level flow of information usually can be characterized in either relative or absolute terms (the heart, for example, though a much smaller organ than the liver, is no less essential to a functioning body). Let us suppose for the moment, as a basis for proceeding further, that we are able to come up with some absolute measure of the information shared among the subsystem units, as envisioned here, of some natural system, and wish to determine whether that array of interactions can be interpreted as a three-dimensional space, as per MDS projection. Any attempt to directly project the set of values will at once be thwarted by the sensitivity of MDS to magnitude trends in the original data: for example, if the first vector of the (similarities) input is composed of small values as compared to the whole rest of the matrix, that element will usually appear in the output configuration as a point that is further from the origin than the others.

To combat this problem, the raw input matrix can first be entropy maximized. This is commonly done in regional science studies to compensate for matters of unequal absolute flow volumes, for example in regional studies of commuter rates among sources and destinations of varying-sized populations [26]; also, the more or less commonly, in other fields. In the case of the regional data, the entropy-maximized scores are then regressed against the original flows data, identifying residuals which are then subjected to further explanatory analysis (usually involving distance-between-locations surrogates that seek to account for the magnitude of the residual).

Just to be clear, in the present simulation exercise MDS is used to identify whether certain entropy-maximized input-output matrices might consist of sets of values that project as a (stress-free) three-dimensional space; that is to say, to examine whether there could be a universe of real world conditions in which subsystemization per se corresponds to three-dimensionality. Its intent therefore does not involve data-mining, as is normally the case with MDS studies. Rather, we are asking a more profound question: might there be a class of internal matrix relations (conceived as flows, similarities of structure, etc.) with numerical characterization qualities capable of signaling three-dimensionality?

2.2. Simulation Results

Simulations from 2014 and 2012 [27] [28] established that only 4×4 element matrices could generate multiple results projecting a three-dimensional space as here described (2×2 matrices produced trivial results; 3×3 matrices only one solution form, thus being unable to account for any kind of system change; 5×5 and 6×6 matrices never yielded stress-free Euclidean projections). Here we therefore focus on 4×4 matrices, and a new, more detailed, analysis.

As posited here, the physical world is (entirely) comprised of natural systems (such as those described through the "conatus" concept alluded to earlier) all of which have subsystemized in a fashion sponsoring subsystem-level inter-/intra-flows of information/energy. The key notion: such flows/structures are posed to be balanced (ultimately, through evolutionary feedbacks) in a fashion resulting in our three-dimensional milieu. While it may sometimes be possible to metrically characterize such inter-subsystem flows (or structures implying flows) directly, such characterizations may or may not directly measure system-level balances. Here, we allow as a given that all four of the posed subsystems contribute equally to an entity's integrity, so any matrix of values describing such a balance must as well. The earlier discussed problem of absolute vs. relative measurement may be overcome by maximizing the entropy of the matrix, as suggested above. One way of accomplishing this is through bistochastization (aka "double standardization" (see [29]-[33])), an iterative operation in which the rows and columns of the matrix are alternately converted to z-scores through as many iterations as it takes to converge to a stable set of z-score values (in one example, "…counties vary widely in population sizes. To control for this (marginal) effect, one may biproportionally/iteratively adjust the row and column sums so that they all converge to be equal (say to 1) … the entries of the doubly-stochastic table provide maximum entropy estimates of the original flows, given the constraints on the row and column sums" ([31], p. 2)). To illustrate, the input matrix of similarities (actually, spatial autocorrelation coefficients) in the following example:

1.868	1.408	1.044	0.962
1.408	1.508	1.275	1.132
1.044	1.275	2.133	2.040
0.962	1.132	2.040	4.795

becomes, after about 2808 iterations of double standardization, the following z-scores:

1.498	0.306	−1.039	−0.765
0.306	1.498	−0.765	−1.039
−1.039	−0.765	1.498	0.306
−0.765	−1.039	0.306	1.498

In this particular example, the matrix of resulting z-scores, when operated upon by a metric MDS algorithm, produces a three-dimensional solution accompanied by zero stress-that is, it represents a faithful geometric projection of the original z-scores data. Note that although all four subsystems' four scores (relative to itself and the three others) are identical as a group, it appears that the relations between some pairings are stronger than for others (*i.e.*, the off-diagonal scores are not all the same value). This reflects a condition of variation whose importance is dealt with below.

Again, the basic idea behind the simulations was to determine whether, and/or how often, 4 × 4 matrices of random numbers, when double-standardized, could produce output matrices of *z*-scores that project as a zero-stress, three-dimensional, space. A million 4 × 4 unconstrained matrices were populated with six-digit random numbers (using R's runif command to produce the strings of random numbers), *z*-scores convergence criteria set, and results tabulated. This effort was an attempt, in effect, to examine a representative sample of the full universal range of possible inter-/intra subsystem interaction states.

As shown in Table 1 and Table 2 and Figure 1 and Figure 2, out of the million test matrices, 250 exceeded the iterations cutoff value (up to 500,000 allowed), and were eliminated from further consideration. Of the remaining 999,750 operations, 4411 (or about 0.4412 per cent) passed the test of producing a symmetric *z*-scores matrix that projected as a stress-free, symmetrically-weighted three-dimensional MDS configuration (the projections were generated by R 4.4.3. software's cmdscale command for metric MDS, backed by the isoMDS command for non-metric MDS). All of the "successful" 999,750 operations passed: 1) the 10^{-11} standard for identifying overall matrix values convergence, and 2) the 10^{-7} standard for reaching *ij*/*ji* element symmetry. Additional standards were investigated—specifically, extending the number of iterations involved, and reducing even further the convergence standards—but these had no substantial effect on the results, while in some instances greatly increasing computer time. The small size of the input matrices (4 × 4) eliminated any concerns over the possible generation of local minima solutions, or even of a difference between the application of metric and nonmetric MDS models.

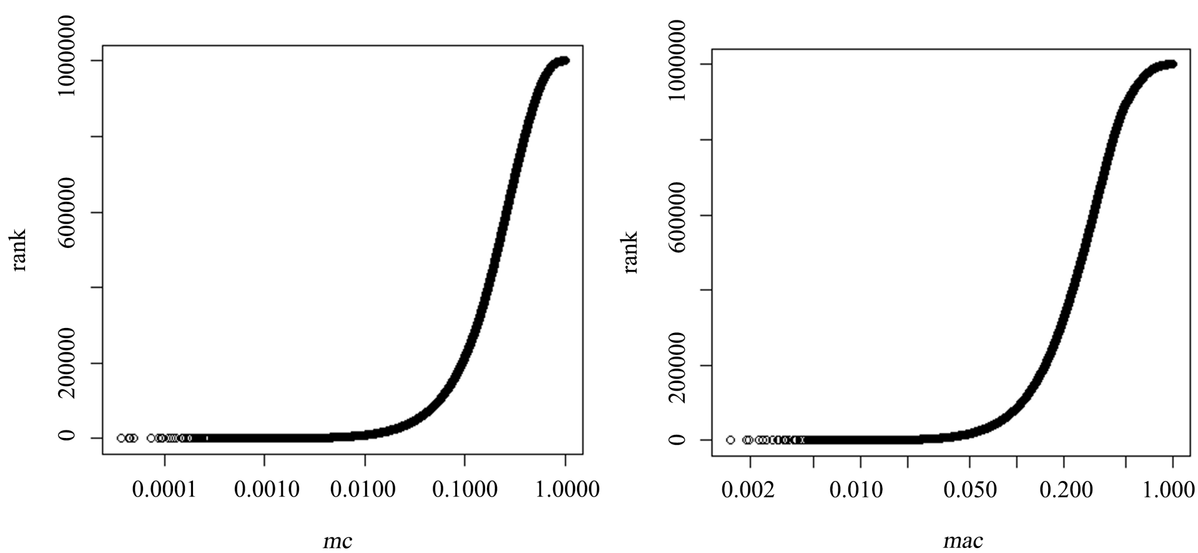

Figure 1. Graphic representation of the 999,750 (of one million) individual z matrices from Table 1 by individual case, plotting the values of correlation means *mc* and absolute correlation means *mac* against their corresponding ranks (with lowest value setting highest rank).

Table 1. Breakdown of the one million *z* matrices tested here, arranged by groupings of ranges of correlation means *mc* and absolute correlation means *mac*.

Ranges of correlation mean *mc*	No. of cases (including cases that reach max iteration)	No. of cases (excluding cases that reach max iteration)	Mean correlation between column vectors of input data and *z*-scores	Population SD of correlation between column vectors of input data and *z*-scores	Mean number of iterations	Population SD of number of iterations	Mean data value	Population SD of data value	Total reaching max iterations
≥0.25	416,373	416,263	0.6342958	0.1754057	888.3229	7912.3741	0.4998617	0.07232083	110
0.125 - 0.25	300,306	300,225	0.7275969	0.1385582	858.8358	7806.0094	0.4998647	0.07172658	81
0.05 - 0.125	193,991	193,950	0.7648669	0.1307789	831.9207	7263.8638	0.4998391	0.07244979	41
0.025 - 0.05	54,466	54,457	0.7806543	0.1276625	869.2057	8079.5412	0.4995075	0.0725005	9
0.01 - 0.025	25,428	25,422	0.784885	0.1272666	881.6041	7938.2897	0.5000219	0.07157573	6
0.005 - 0.01	5984	5983	0.7867168	0.1282954	1002.653	8604.9279	0.4995759	0.06988527	1
0.001 - 0.005	3112	3110	0.7875569	0.1271532	935.2868	10178.113	0.5012355	0.06883946	2
0.0005 - 0.001	222	222	0.7756782	0.1356939	856.6667	3792.5287	0.5062126	0.07065267	0
0.0001 - 0.0005	109	109	0.786483	0.1276445	879.2936	4276.3209	0.5047648	0.07018308	0
<0.0001	9	9	0.8068136	0.1356075	198	169.1988	0.5090597	0.05004942	0
Total	1,000,000	999,750							250

Ranges of absolute correlation mean *mac*	No. of cases (including cases that reach max)	No. of cases (excluding cases that reach max)	Mean correlation between column vectors of input data and *z*-scores	Population SD of correlation between column vectors of input data and *z*-scores	Mean number of iterations	Population SD of number of iterations	Mean data value	Population SD of data value	Total reaching max iteration
≥0.25	544,104	543,951	0.651922	0.169805	870.6088	7805.4	0.4998699	0.07185531	153
0.125 - 0.25	319,493	319,425	0.75223	0.135148	829.4897	7397.219	0.4997728	0.07274028	68
0.05 - 0.125	118,402	118,378	0.776263	0.13137	909.9709	8311.47	0.4999765	0.07218386	24
0.025 - 0.05	14,720	14,718	0.775471	0.131043	1185.024	10007.08	0.4993156	0.06966983	2
0.01 - 0.025	3010	3007	0.766263	0.138582	1235.554	8453.972	0.5013866	0.06779061	3
0.005 - 0.01	238	238	0.75423	0.14428	2078.151	10830.22	0.5000683	0.06704983	0
0.001 - 0.005	33	33	0.774129	0.134377	391.2121	656.2436	0.4875596	0.06003254	0
0.0005 - 0.001	0	0							0
0.0001 - 0. 0005	0	0							0
<0.0001	0	0							0
	1,000,000								250

Table 2. Breakdown of the 4411 z matrices that successfully project as three-dimensional systems, arranged by groupings of ranges of correlation means *mc* and absolute correlation means *mac*.

Ranges of correlation mean *mc*	No. of cases	Mean correlation between column vectors of input data and *z*-scores	Population SD of correlation between column vectors of input data and *z*-scores	Mean number of iterations	Population SD of the number of iterations	Mean data value	Population SD of data value
≥0.25	1957	0.63476	0.180106	2433.541	12607	0.500183	0.0718314
0.125 - 0.25	1262	0.72288	0.140065	2299.952	16299.4	0.498635	0.0717285
0.05 - 0.125	822	0.75199	0.132241	1780.007	6818.241	0.494676	0.0695772
0.025 - 0.05	219	0.79359	0.129007	1733.808	5992.249	0.504258	0.0672722
0.01 - 0.025	104	0.80901	0.119118	5485.692	24234	0.505365	0.0692118
0.005 - 0.01	29	0.7563	0.111193	2031.276	5710.56	0.477092	0.0811835
0.001 - 0.005	17	0.88891	0.056924	8759.471	31671.17	0.53393	0.0729635
0.0005 - 0.001	1	0.90499		261	NA	0.396651	
0.0001 - 0.0005	0						
<0.0001	0						
	4411						

Ranges of absolute correlation mean *mac*	No. of Cases	Mean correlation between column vectors of input data and *z*-scores	Population SD of correlation between column vectors of input data and *z*-scores	Mean number of iterations	Population SD of the number of iterations	Mean data value	Population SD of data value
≥0.25	2429	0.64776	0.173672	2263.722	11528.7	0.50089	0.07131
0.125 - 0.25	1363	0.73954	0.140257	2325.445	16117.72	0.495521	0.0714459
0.05 - 0.125	517	0.77974	0.122626	2256.418	11413.64	0.499428	0.0709508
0.025 - 0.05	74	0.8312	0.132938	3778.811	12365.89	0.497289	0.0718926
0.01 - 0.025	24	0.79594	0.134438	1480.958	1849.801	0.505374	0.0590136
0.005 - 0.01	4	0.84421	0.1012	34133.5	58436.34	0.467017	0.0389741
0.001 - 0.005	0						
0.0005 - 0.001	0						
0.0001 - 0.0005	0						
<0.0001	0						
	4411						

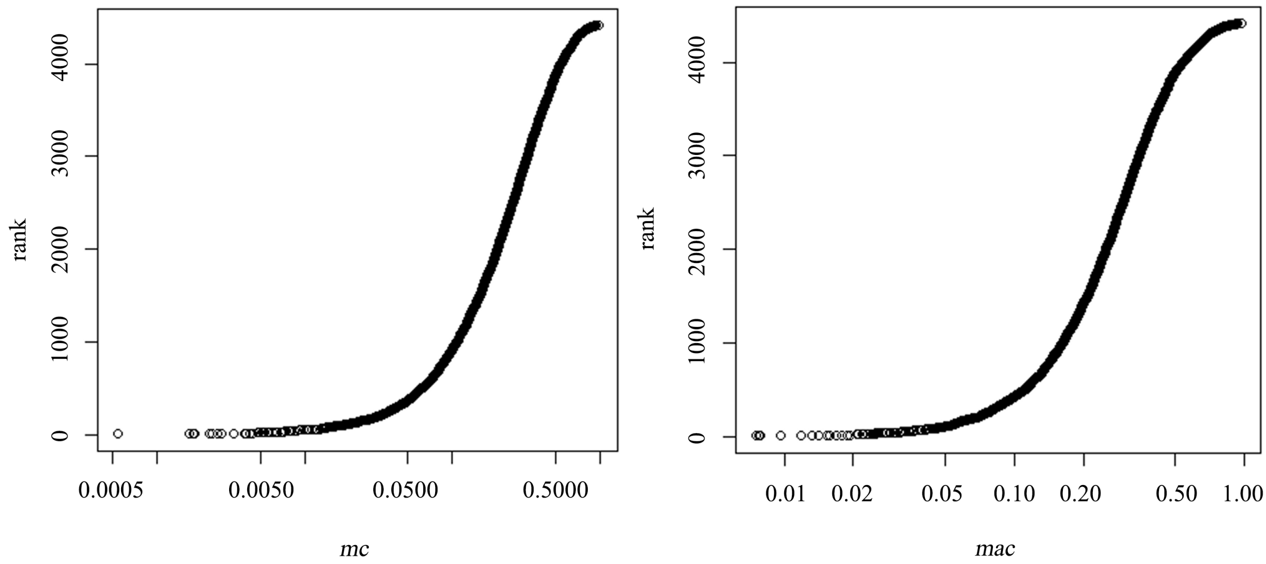

Figure 2. Graphic representation of the 4411 individual z matrices from **Table 2** by individual case, plotting the values of *mc* and *mac* against their corresponding ranks (with lowest value setting highest rank).

Importantly, the 4411 sets of input that produced "passing" solutions exhibited highly varying internal correlation levels among their component vectors (**Table 2** and **Figure 2**). This redundancy is evident through an examination of the symmetric Pearson *r* correlation matrices derived from each 4 × 4 data set input. For any such correlation matrix, the lowest mean *r* value that can be obtained for the sum of its elements is zero; anything higher than that indicates some overall level of correlation among two or more of its component vectors, an outcome we are referring to as its net redundancy. Two such measures of overall level of redundancy were calculated for each matrix treated: 1) the simple sum of the vector means in the correlation matrix, divided by four (*mc*), and 2) same, but more conservatively employing the mean of the absolute value of the vector means, deriving a "deviation from zero" kind of summary appraisal (*mac*).

An examination of these results leads to four suggestive observations that potentially link them to real world characteristics of evolutionary change.

First, the proportion of cases (0.44 percent) that pass the spatial projection test seems, subjectively, to reasonably fit what one would expect for a real world condition in which persistence of natural structure is highly constrained, specific, and probabilistic. At the very least, were either none or all of the 999,750 trials successful in producing a projectable zero-stress array of *z*-scores, further discussion on the subject would be pointless. Additionally, the simulations discussed in [27] and [28] also examined a number of other initial 4 × 4 matrix conditions (e.g., beginning with random numbers arrayed such that all *ij* values equaled *ji* values, and performing analyses on spatial autocorrelation coefficients describing nearness relations on grouped elements of two- and three dimensional point patterns), and all of these yielded various small to moderate proportions of instances that passed the spatial projection test as well (in fact, and additionally interesting, the proportions were higher for two-dimensioned systems, and higher yet for three-dimensioned ones).

Second, it is apparent from Table 2 that a wide range of mean redundancy values (again, mean correlation statistics *mc* and *mac*) can be associated with matrices that actually project as stress-free configurations in MDS. In the real world, it is well known that most systems undergo changes between their incipience and maturity periods that involve switching over from an early emphasis on growth (and greater redundancy of practical function) to a later one more emphasizing efficiency of operation; this has been noted in numerous studies (and in various ways) dealing with, for example, ecological succession, trends in individual organism maturation, and stream basin evolution. Had the simulations here not uncovered such a flexibility in the results, it would be difficult to make any kind of argument favoring their application to understanding a real world evolutionary process.

Third, and equally important, a quick perusal of Table 3 shows that the twenty-five very least internally-redundant matrices do not appear to be converging on a single particular configuration of *z*-scores (see Supplementary Materials 1 for more complete tallies). For example, the matrix with vector values of 1.505/0.221/−0.550/−1.176 produces an *mc* (mean correlation) value of 0.001792, whereas the next-ranking one, with an *mc* value of 0.001751, is generated from the much different *z*-scores set 0.840/0.582/0.276/−1.697. In this simulated space, therefore, a wide range of relative magnitudes of similarities (conceived here as distances) is identified, a condition much easier to connect to real world conditions of vastly different kinds of systems than had a single pattern of convergence to zero redundancy turned up. (A three-dimensional view of the simulated configuration of the second matrix mentioned above is given in Supplementary Materials 2).

Table 3. Data for the 25 matrices (of the 4411 successfully spatially-projecting *z*-scores) producing the lowest redundancy (*mc*) values.

Rank	*z*-score				*mc*	*mac*
25	1.1328	0.7325	−0.4633	−1.4020	0.006437	0.029067
24	1.6769	−0.3371	−0.3726	−0.9672	0.006342	0.029969
23	1.2755	0.5523	−0.4681	−1.3597	0.006065	0.035946
22	1.5401	−0.0300	−0.2616	−1.2485	0.005696	0.044499
21	1.0377	0.8543	−0.4968	−1.3952	0.005378	0.018777
20	0.9662	0.6767	−0.0280	−1.6149	0.005224	0.037503
19	1.1227	0.8095	−0.6361	−1.2960	0.005186	0.015425
18	1.4231	0.4189	−0.6944	−1.1476	0.004916	0.007854
17	0.8439	0.5947	0.2553	−1.6938	0.004871	0.035097
16	1.0944	0.6082	−0.1502	−1.5524	0.004805	0.032949

Continued

15	1.5865	0.0680	−0.5928	−1.0617	0.00478	0.034008
14	1.0390	0.6470	−0.1080	−1.5780	0.004433	0.028798
13	1.3049	0.6351	−0.8933	−1.0468	0.004288	0.011923
12	1.4541	0.3357	−0.6022	−1.1876	0.003995	0.023027
11	1.5565	0.1339	−0.5895	−1.1009	0.003895	0.040988
10	1.5050	0.2469	−0.6121	−1.1398	0.003892	0.020753
9	1.3141	0.5469	−0.5823	−1.2787	0.003313	0.04265
8	0.9748	0.5629	0.1117	−1.6494	0.002711	0.026421
7	1.3231	0.6128	−0.9571	−0.9787	0.002542	0.009724
6	1.3508	0.2391	−0.1412	−1.4486	0.002375	0.038502
5	1.3841	0.5181	−0.8935	−1.0088	0.002278	0.024461
4	1.5050	0.2214	−0.5503	−1.1761	0.001792	0.029031
3	0.8396	0.5819	0.2756	−1.6972	0.001751	0.022093
2	1.2871	0.3690	−0.1815	−1.4746	0.001648	0.019221
1	1.3999	0.4416	−0.6473	−1.1942	0.000538	0.014109

Lastly, and most important of all, from the variety of matrices and mean redundancy values obtained, it seems likely that many or most particular sets of *z* matrix values that produce a stress-free projection can, when altered slightly, produce another such solution that exhibits slightly less (or more) redundancy. Probably, a large (but varying) number of such divergences are possible from any given starting point, and this arguably mirrors the way structures change over time in the real world. Note that "back-sliding" to a less efficient/*more* redundant structural form is also possible here, and this also happens in the real world, as when an individual biological entity falls ill, or when a stream basin experiences an equilibrium-disturbing rejuvenation event after a sudden lowering of basal sea level, or stream piracy episode.

The simulations as described thus provide support for the basic philosophical model discussed here. The simulation methodology is quite straightforward, employing techniques with long histories of application in a large variety of contexts. The particular convergence cutoff values chosen have some effect on the exact results, but the basic point is made whether 4411 "successes" are obtained, or some other similar total: there turn out to be a limited but nontrivial number of 4 × 4 input-output matrix formulations that can define a three-dimensional Euclidean space.

It must be remembered here that the goal of the simulation was to determine whether some small (or larger) proportion of randomly-generated matrices, projected as distances, could achieve the distinction of defining—not any *particular* geometric figure—but instead, three dimensionality itself. The results obtained may be expected to vary slightly as different-sized samples of constrained or unconstrained random numbers, alternate software algorithms, and/or varying algorithm cutoff values are applied (as is evident from studies [27] [28], whose simulations, employing different algorithms and constraints, yielded very similar outcomes), but the basic point seems quite robustly made.

It of course cannot be claimed that a strictly numerical approach of this kind to the conceptual matter at hand in any way proves that physical space self-organizes on the basis suggested. Yet it does provide an argument for not dismissing the notion that it might: had the exercises described determined there were no initial matrices of values that yielded zero-stress three-dimensionality (or that they all did), further explorations concerning the subject would be pointless. Moreover, there would be little reason to make more organized attempts linking the conceptual and statistical meanings of some of the terms used here (e.g. "information", "subsystem", "correlation/redundancy")—at least, in this vein.

Additionally, it suggests a foundation for possible forms of real world-based empiricism. The papers mentioned earlier ([27] [28]) enter into that subject, and next can be briefly considered, along with the results of a few additional pilot studies.

3. Discussion: Related Empirical Work

Under ideal circumstances it might be possible to directly quantify the information transfer properties among real world flow units per the subsystemization scheme proposed here, but usually, at least (especially for highly organized systems such as organisms, or highly changeable ones such as stream systems) no appropriate means of direct measurement of ongoing process may exist. Even in such cases, however, influence of the type imagined might well show itself in the form of *results*; *i.e.*, as structured patterns of secondary effects. (A parallel example of this kind of thinking is manifest in the existence of faunal regions; no single measurable flow of information is apparent, yet the differences among such units can be explained as a result of the interplay of many large scale, long-term, influences.) In some cases, the influence involved could be singular; for example, dominantly the result of gravitational forces. For this reason, the first two attempts to identify the possible existence of such an effect came with geophysical systems.

In [28], thirty-three small stream basins in Kentucky were examined to determine whether there was any evidence supporting the hypothesis that their surfaces had been shaped according to a differentiation into four functional zones of elevation whose interactions with one another as dynamic subsystem units were yielding the observed surface topographies (largely, through an interplay of the potential energies inherent in varying elevations above sea level). Stream basin surfaces evolve probabilistically according to changeable internal and external influences, yet in general tend to move toward a lower-energy equilibrium characterized by a balance between (small amounts of) erosion and deposition. In the present context it was thus expected that a spatial autocorrelation analysis of elevational zones would produce input-output *z*-scores passing the spatial projection test, and yielding a range of *mc* and *mac* statistics corresponding to variations in their average surface equilibrium conditions. Sample grids were laid over the test basins and the elevations sampled in each basin (from ArcGIS platform data), then as sets subjected to nonhierarchical cluster analyses. This was carried out for subsystemizations into $n = 2$, 3, 4, 5 and 6 units, and at three finenesses of spatial sampling density. The results are discussed in detail in the original work, but in brief it can be mentioned here that 30 of 31 of the spatial autocorrelation matrices, once transformed into *z*-scores, did in fact project as a zero-stress three-dimensional space, and there was some considerable variation in the redundancy statistics that were highly correlated with unevenness of surface. (In a previous pilot study a separate measure of basin slopes variability, though fairly primitively constructed, accounted for some fifty percent of the variation in that set of twenty-five basins' redundancy statistics; the same test was unfortunately not applied in the 2012 [28] study). The three pertinent summary figures from [28] have been grouped together as **Supplementary Materials 3a, 3b and 3c** here. This is a must-see; note in those materials the results for the four-subsystems analyses in the third figure, which clearly provides evidence that under those conditions there is a causal influence involved that is lacking in the 3-, 5- and 6-subsystem groupings.

In [27], a system with rather different internal differentiation properties was examined: our Earth. In this instance we have a system with relatively distinct and permanent internal boundaries that has been evolving as an integral unit for more than four billion years. Such a system might be expected to have evolved a permanent internal configuration, and in fact Earth is internally subdivided into the inner core, outer core, mantle, and dynamically active surface zones. Sample grids of about fifteen to twenty thousand points each were superimposed three-dimensionally over the Earth's interior, the points falling into each of the four zones collected together, and 4×4 spatial autocorrelation coefficients matrices derived from among these groupings. A number of slightly different samples were taken, including ones using published estimations [34] of two periods from the distant past exhibiting greatly differing surface continent positions as compared to the present. All of the initial spatial autocorrelation matrices constructed yielded zero-stress spatial projections via MDS; interestingly, the accompanying summary redundancy statistics were all very low, in the range of 0.0011 to 0.0035. This is as low as the lowest values (**Table 3**) obtained in the simulations described earlier—that is to say, among nearly a million analysis runs. These results are just as might have been expected for such a well-evolved, "highly improbable", system. Still, an updated analysis, bringing to bear an even finer sampling net and state of the art appreciations of varying crustal depths and internal zones boundaries, would likely lower the *mc* and *mac* values even further.

Though in the main concerning geophysical realities, neither of these analyses involve systems that are completely abiological, as the existence of life on the surface of our planet produces a highly significant moderating/shaping effect on its active surface zone. In addition to this published work, several other tests of the idea have been attempted in "pilot study" fashion, and may now be quickly mentioned. (These studies, along with the results of some additional simulation efforts, are explained further at an informal html site that has been set up at https://people.wku.edu/charles.smith/once/writings.htm#2.)

In the first such study, a maximally-spaced sample of 325 estimated elevations above and below sea level along the Earth's surface was taken, and submitted to the same kind of spatial autocorrelation analysis as described earlier; three variations on the theme all proved successful (or nearly successful) in passing the spatial projection test. These results were encouraging, considering the relative crudeness of the measurement effort.

Another such test on patterns was applied to a sample of butterfly images—specifically, to a number of species that exhibited wing coloration patterns comprised of four different colors. Again, though the measurement effort involved assumptions, a fairly impressive percentage of the forms exhibited patterns passing the spatial projection test.

A more extensive analysis was applied to human population patterns in the Indianapolis, Indiana region in the Midwest of the United States. Here, population levels of the nearly 400 townships across the region were retrieved from decennial federal census reports over the period 1850 to 1980. Again, populations for each year were clustered into four groups, and inter-/intra-group spatial autocorrelation analyses administered. While the first ten systems (1850 to 1940) passed the spatial projection test, the last four did not: quite possibly because place of residence alone was no longer a valid measurement of system function, which after 1950 was not taking into account the differing locations of home and place of work. (Indeed, adding such an adjustment could provide a further test of the concept, or even predictions of future population distribution trends.)

Lastly, an attempt was made to model four element-constituted amino acids on the basis of their bonding patterns. Again, crude measures were applied, but on the whole, encouraging results were obtained. It would take a professional chemist to put together a more definitive appraisal.

4. Conclusions

The model presented here projects some initially difficult-to-appreciate concepts, but it also has the merit of being relatively easy to explore (not, for example, requiring years of expensive laboratory work or field studies—or perhaps, even grant support altogether). One might complain that it doesn't focus on immediate causalities, but the history of science is strewn with similar kinds of discoveries that have proved useful; *i.e.*, observed patterns that are suggestive of process, but were only later linked to more specific mechanisms (think, for example, of the contributions of Kepler, or Darwin/Wallace).

One question that may reasonably be asked has to do with the four-element subsystemization pattern studied in the simulations reported here, and in [27] and

[28]: is this something that should have been expected, after all? Two good reasons can be given for thinking so, *a priori*. First, it is well known as an element of basic geometry that it takes a minimum of four coordinate nodes to specify a three-dimensional space. Second, equations representing the functions of systems involving five or more variables are almost never solvable, making it possible to imagine how a four-subsystem structure might be better able to self-adjust—that is, to reduce its internal redundancies through the operation of various feedbacks attuned to this kind of solution.

Meanwhile, studies on other natural patterns might lead to immediate benefits, regardless of knowledge of details of causality. For example, Alzheimer's disease is characterized by a degeneration of several basic brain functions, and these are spatially evident in pattern changes of blood flow, temperatures, electrical activity, etc. It could be the case that these patterns can be identified at a much earlier stage through the methods described here, aiding in delay or even prevention of onset of the condition. And many natural structures are already known to have special relations to the number four, among these the number of bases in the DNA molecule, the frequency grouping character of brainwaves, and even the fundamental interactions of gravity, electromagnetism, and strong and weak nuclear forces in physics. Perhaps some of these structural peculiarities are only awaiting a new context for further applications.

Indeed, additional applications of the concepts discussed here may be possible in fields ranging from climatology and meteorology to regional planning, and even, in reverse-engineering fashion, to mechanical and electrical engineering. Such work will necessitate input from variously-trained professionals, of course, and usually require specialized and/or proprietary data sets, but the exploratory programming involved in many instances may not actually be very difficult.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Supplementary Materials

1. MDS records for the twenty-five *z*-scores matrices that passed the spatial projection test, while producing the lowest redundancy statistic *mc* scores.

2. Visual (3d) representation of the MDS projection with the 8th lowest *mc* score.

3. Three figures from [28] describing the results of that analysis, concerning the topographical character of thirty-three stream basins in Kentucky.

a. Summary of spatial projection simulations for matrix configurations of dimension 3 × 3 through 6 × 6. Circled numbers refer to data at the right margin giving the number of simulations in each test and the standard deviations accompanying the mean values plotted. The plotted numbers are the two mean "redundancy" [in the present paper, *mc* and *mac*] values obtained for each classification grouping. Colored lines are for readability purposes only. Point values 17 through 20 are compiled from subsets of the data leading to point values 2, 6, 10, and 14, respectively, and are for those matrices that passed the spatial projection test (nos. 14/20 and 6/18 parallel the 4411 that passed the test in the present study). Note the very smooth functions of decrease in *mc* and *mac* values across the four formulations.

b. Summary of variation-explained statistics obtained for the nonhierarchical clustering of the stream basins elevation data (vectors) into two through six classes, for three original spatial sampling densities. The plotted points are the mean (n = 31) variations-explained for each classification. Colored lines connecting points are for readability purposes only. Note the very smooth function of increase in the variation-explained statistic.

c. Summary of the spatial subsystemization properties of topography in 31 Kentucky stream basins, based on three spatial fineness levels of sampling. The plotted values in the top three sets of four points correspond to the *mac* redundancy statistics of the present work; those in the bottom three, the *mc* statistics. The associated standard deviations are written out next to each plotted value. Colored line coding connecting points is for readability purposes only. The results as displayed here, showing significant reductions at the four-class level, are in marked contrast to the smooth simulation and clustering results shown in A and B, strongly suggesting that the spatial (topographic) expression of the basins is causally related to a functional subsystemization process operating at the four-class level.

The supplementary materials can be found at https://docs.google.com/spreadsheets/d/1q2cmz930x3J3d2XW_Rc1aalE5qqhUDBF/edit?usp=sharing&ouid=116218283108784116415&rtpof=true&sd=true.