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A GENERAL APPROACH TO THE STUDY OF SPATIAL SYSTEMS

I. The Relational Representation of Measurable Attributes

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In this, the first part of a two part work, a general model of spatial organization is introduced. Following a brief synopsis of some of Spinoza's and Leibniz's views regarding natural structure, an extension of the Spinozian model is presented in which the attribute spatial extension is portrayed as a relational system that implicitly underlies the differentiation of sensible space into "modifications" ("natural systems") and the latter's subdifferentiation into "modes." On the basis of this model, all instances of modal differentiation are understood to take place in a manner explained by this relational structure, the existence (but not the specific characteristics) of which is initially assumed. The nature of the structure is then deduced according to a "most-probable-state" kind of logic; next, it is demonstrated via simulation that the resulting aspatial model of internal relations has a corresponding spatial interpretation (and therefore, in theory, that sensible space structures can be supported by the particular rational ordering posed). The matter of how to apply the model to the study of real world systems is taken up last; discussion focuses on related aspects of the treatment of equilibrium and nonequilibrium systems and the recognition and measurement of modal structures.

INDEX TERMS: Spinoza, Leibniz, Rationalism, spatial extension, relational system, spatial systems, most-probable-state, hierarchical class, entropy minimization, entropy maximization.

INTRODUCTION

Although the systems approach has become firmly entrenched in our current way of thinking, it can hardly be denied that we are a long way from having a truly general theory of spatial systems at our command. Not only do we have no comprehensive model or set of models that can be used to interpret spatial order, but also lacking is a general philosophical position within which practical investigations might be related to their more universal context. The work that follows attempts to address these issues at a very fundamental level. In it is outlined a philosophical approach to spatial systems organization which, when extended to the level of operational modelling, provides both descriptive power and conceptual flexibility. The discussion is developed in two parts. Part I begins with an extension of some Rationalist philosophy concepts intended to provide a more useful basis for the study of spatial systems. Following this is the presentation of a model of internal relations consistent with a "most-probable-state" kind of logic and earlier-stated goals. Lastly, the matter of how to apply the model to real world situations is taken up. In Part II, the overall approach is considered in terms of its potential for aiding the study of geographic systems; this second section features both general discussion and two specific instances of application.

SOME ASPECTS OF THE PHILOSOPHY OF SPINOZA AND LEIBNIZ

While Hegel (1770–1831) is often credited with laying the philosophical cornerstone for modern systems thinking with his doctrine of internal relations, I believe it is more productive to first turn one's attention to the seventeenth century philosopher Benedict de Spinoza (1632–1677) for the most profound treatment of nature as a general system. This is rarely done now, though Spinoza's ideas have historically had considerable influence on a number of notable thinkers (for example, Leibniz, Bergson, and Teilhard de Chardin). I shall not take the time here to attempt a defense of Spinoza's ideas as regards those of other philosophers but instead will plunge directly into a brief synopsis of concepts relevant to the present discussion.

Spinoza is often considered quite modern in his views; in theory, at least, his method of inquiry reduces to a dialog between investigator and nature that is virtually devoid of metaphysics. Nonetheless, he was a firm believer in the existence of God. Spinoza's God, however, was inseparable from existence itself. This then-heretical pantheism was consistent with his view that nature as a whole constituted a kind of all-encompassing continuum he referred to as "substance." It was impossible for man to obtain any direct knowledge of substance, which existed under some number of "attributes" through which, one might say, its intrinsic order was expressed. Importantly, man has no direct sensible knowledge of these attributes either; instead, each is implicit in innumerable "modifications" which we *can* directly perceive and to which we attach various kinds of meaning. This point must not be confused: while it is our habit to think of the things that "fill space" as having properties such as color and weight that directly characterize their natural essence, Spinoza did not see things in quite this fashion. According to his way of thinking, a red rose is not an object with the attribute of redness; rather, it constitutes a finite mode extending "redly" under the attribute of spatial extension. This line of reasoning can be extended to the consideration of "spatial distribution" itself; the distribution of roses can thus be looked on as a modification of the attribute spatial extension in the same way an individual rose is. Note, moreover, that measures describing the "spatial distribution" of roses do not necessarily give us more information about the attribute spatial extension than do, say, measurements taken on their anatomical structures. On first contact this curious turn of affairs seems rather unhelpful in clarifying the meaning of "spatial extension." It is not, however, because it focuses our attention in new and useful directions. Most importantly, it forces a critical re-evaluation of the tacit assumption that things "fill space." The Spinozian understanding of nature is instead one in which all the directly manifest properties of extension represent patterns of interaction underlain by common rules of organization referable to a more fundamental ordering process. For the student of spatial systems, this philosophical position holds great appeal. An appropriate model of that process might suggest methods that could be applied in the study of virtually any system of interest.

Spinoza's model of natural order is schematically diagrammed in Figure 1, where S represents substance, A_i represents attributes, and M_{ij} represent modifications. Note the branching structure involved: substance is both unique and all-inclusive and subsumes all attributes, to which in turn are referred unique assemblages of virtually infinite numbers of modifications.

Though extremely brief, the preceding description of Spinozian ideals forms an

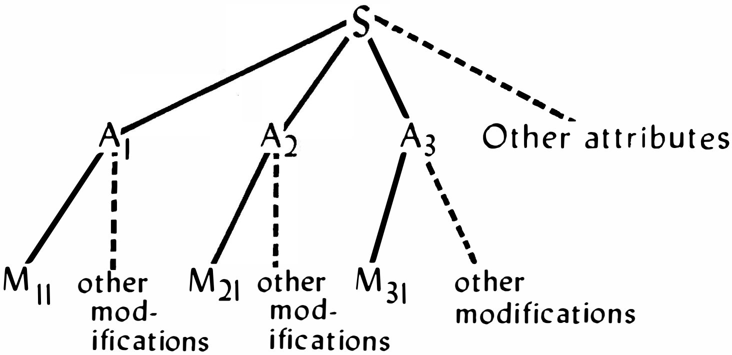

FIGURE 1

adequate base for what follows, which is not intended as an analysis of that philosopher's work.[1] Spinoza's ideas do, however, provide a useful point of departure for the discussion. Though appealing as a philosophical structure, the Spinozian view of nature contains a fundamental weakness that renders it difficult to apply to the study of real world situations. Spinoza apparently believed that it is impossible to isolate specifics from the continuum of reality; that is, each modification of an attribute is in essence both unbounded and unrelatable to other modifications. This interpretation of nature seemingly casts it as having order but no parts, or at the least no independently meaningful parts. This emphasis on "vertical" organizational principles makes it difficult to order our impressions of the "horizontal" links that form the major part of our day-to-day existence. The characteristics of interaction of finite entities such as human beings thus remain obscure in Spinoza's writings, or at best are disposed of in a manner not lending itself to practical analysis.

In criticizing Spinoza's beliefs, the German philosopher/mathematician Leibniz (1646–1716) dwelled on the issue of the supposed indivisibility of nature.[2] Leibniz felt that not only was the whole of reality manifest in any given portion of it (as Spinoza also believed), but that these portions should be conceived as individual entities in their own right. This became the foundation for the "monad" concept, which provided for a kind of elementary particle in each of which was reflected the characteristics of all other such particles. In this view was the flexibility necessary for an interpretation of horizontal relationships in nature. It also permitted—as did Spinoza's ideas—hierarchical representation of nature. Nonetheless, Leibniz was unable to develop from this base a satisfactory model of how it was that any given monad actually contributed to a group structure that could mediate such universal symmetry. It is to a consideration of this matter that we first turn in the present study.

AN EXTENSION OF THE RATIONALIST APPROACH

There seems little value in attempting to argue that the basic position of either philosopher is inherently more right or wrong than the other's. Rather, in what follows I have attempted to combine what I see as being critical positive elements of each position in such a fashion as to make possible new directions of study. This has necessitated the invention of some new concepts; hopefully, the ends will be viewed to justify the means.

Let us therefore begin with a few definitions (these will be retained from here on; more colloquial applications of the same terms will be especially noted). For present purposes, an "attribute" will be considered as that in virtue of which substance may be understood. This is in close accordance with Spinoza's definition of the term. Following Spinoza's views on the matter, I accept that while in theory any number of attributes might exist, human powers are limited to the recognition of but two: spatial extension and thought. Here, attention is focused on the development of ideas relevant to consideration of the first. It should be noted, however, that the discussion developed is not irrelevant to the study of the attribute thought;[3] neither is it cast in such a fashion as to conflict with twentieth century ideas regarding the principle of non-independence of observer.

A "modification" will be defined as a (the) sensible aspect of an attribute. This is also a very Spinozian manner of understanding, but contains beyond his view some implicit associations that will only become apparent after the remaining definitions are presented. At any rate, modifications are regarded here as entities occupying complete and delimitable domains—in the case of modifications referring to the attribute spatial extension, sensible (in the literal sense of the word) spatial domains. Examples of the myriad of modifications referable to spatial domains include: the distribution of organisms, cities, continental crust (sial), etc.; cells; and organisms.

We next turn to "modes." These are to be understood as quasi-finite subunits or classes of subunits resulting from the internal differentiation of a given modification (i.e., its natural subsystemization). The implications of this definition are considerable here, because I will suggest shortly that modifications differentiate internally in a manner having regular properties. Examples of sets of modes would include faunal and floral regions, the classes of population centers in the central place hierarchy of urban geography, the individual continental masses, and the organelles and organal systems making up individual cells and organisms, respectively.

The "hierarchical class" is introduced here to name that in virtue of which the modes associated with a given modification are understood/expressed as a whole in terms of one another. Its purpose is thus to facilitate *description* of the commonalities referable to the functional links interconnecting a given system of modes. In particular, it is designed to describe a set of relationships involving *all* modes in a particular modification. By analogy, for example, we cannot expect to have a complete concept of color unless we have some means of relating *all* of its component colors to one another in some internally consistent fashion (for example, by linking them to particular combinations of hue, density, and brightness). Using the same reasoning, neither can we expect to have a coherent understanding of world faunal or floral patterns in the presence of a model that ignores conditions of association between particular geographical units. An analogous argument can be made regarding central place hierarchy modelling. In the case of the distribution of the continental masses, the search continues for a causal model that can account for all the known characteristics of plate movement and interaction. Cells and organisms are living entities whose very survival depends on the complete functional interdependency of their component subsystems. In all these examples, the internal differentiation of the modification in question into modes may be described in terms of associations inherent among the latter; the hierarchical class concept provides a means of referral to such differentiation that highlights these associations.

The last term introduced at this point is "representation." This is to name those characteristics of the internal relational order of attributes that are expressed in a hierarchical class; that is, that prescribe the way modifications internally differentiate *as* modes. We may thereby speak of the "representation" of an attribute in its various modifications. Following Spinoza, I accept that all modifications referable to the attribute spatial extension in some fashion reflect the nature of that attribute. From that starting point, it is natural to consider what the relation between attributes and modifications may be that allows this to be so. In accepting the Leibnizian position that any given modification of an attribute is somehow reflected in any other given modification, we are likewise led to question what it might be that all modifications have in common that could make this possible. In the system presented here, the holistic and "monadic" positions of Spinoza and Leibniz, respectively, are integrated through the use of the notion of representation. Simply, "attribute" is the name given to rules of representation that actualize the infrastructure of modifications. These rules are therefore viewed as being implicit in anything physically extended, forming the basis for all indentifiable hierarchical classes. This solution is consistent with Spinozian reasoning, which leads to the conclusion that there exists but one set of rules referable to the organization of the sensible space though which modifications may be characterized. It also yields the idea that in each modification is reflected the fundamental nature of all other modifications: things cannot be otherwise if, as we assume here, all modifications differentiate into their characteristic observed forms on the basis of the same relational structure. We are thereby provided with a tangible link to Leibniz's "monads." Nonetheless, the structure envisioned remains vertically organized, since one can just as easily understand that which is physically extended to relay information about the nature of its underlying rules of representation as the reverse.

From the above it can be seen that hierarchical classes must be isomorphic with respect to their characterization of inter-modal commonalities. Again, this is a product of the relationship between the structure of the attribute spatial extension and its representation in modal differentiation. This way of viewing the organization of the natural world has profound implications for the way problems of spatial distribution and diversification must be treated. Space itself is no longer viewed as having directly evident (or perhaps better put, "naïvely evident") qualities; rather, it becomes in effect a system of relations of which there can probably be no single absolute rendering. To avoid confusion, it is necessary to recognize two kinds of "space", varieties which may be termed "relational" and "sensible". The former refers specifically to the attribute spatial extension and its conditions of internal relational order. The latter may be taken as the sum of sensible spatial domains occupied by modifications of the attribute spatial extension. Through this separation of terms, the general and observable "facts" of distribution of things—including the notion of "spatial pattern" itself—are referable only to sensible space; the fundamentals of their organization into spatial systems, however, must be interpreted as a function of the characteristics of relational space.

This conceptual separation of space into sensible and relational components will inevitably lead to some initial confusion that must be overcome before these "Neo-Rationalist" concepts can be successfully applied to considerations of real world conditions. Especially troublesome may be the fact that it is possible for two modifications to exhibit the same sensible properties of extension (i.e., have the same spatial domain) but be comprised of modal systems of entirely different areal differentiation. An example related to some of my own work on zoogeographic regions may aid in appreciating this point.[4] Suppose we are considering the two modifications "terrestrial mammal distribution" and "terrestrial animal distribution." The spatial domains of these two modifications are essentially identical for the reason that virtually all exploitable terrestrial habitats that exist are inhabited by both mammals and animals in general. We must not jump to the conclusion, however, that the system of modes (regions) into which each has differentiated to maintain an internal relational structure consistent with that of the underlying attribute spatial extension need be areally identical. The biogeographic histories of the two groups are not identical (mammals, of course, having evolved much later than animals taken as a whole), and must be viewed as responses to generally different sets of proximate causal influences when viewed within their sensible space context. Nonetheless, according to the understanding being developed here we can still believe that these two histories have devolved from a single relational ordering process implicit in the attribute spatial extension. In so doing, we attribute to nature the capability of producing structures that are unique with respect to their characteristics of occupation of sensible space, but isomorphic with respect to their fundamental organizational properties.

A related candidate for confusion is the difficulty that it seems particular modes might be associated with more than one modification. For example, though we might determine "Australia" to be a region (mode) within a faunal regions system (hierarchical class) depicting mammalian distribution (a modification), one might argue that this areal unit also hosts non-mammalian faunal elements and thus should be considered to contribute, *ipso facto*, to understandings of the organization of other world faunal distribution patterns. Recall, however, that a modification and its modes can be described in no other way than through a hierarchical class, and that the latter is defined here in terms of, and as reflecting, the structure of relational space, not sensible space. It is therefore immaterial that Australia may be "in the same physical place" with respect to two modifications; what is significant is whether it is expressive, as a sensible spatial (and natural) unit, of the relational structure of each (of course, a given sensible space unit such as Australia *could* qualify as a mode in more than one hierarchical class; the point, however, is that it does not *have* to).

Neither can we view modifications of small spatial domain as in some fashion being "enclosed by," or existing as a subset of, modifications of larger spatial domain. The spatial domain of the distribution of the class Mammalia, for example, is much larger than is the spatial domain of the distribution of any given mammal species, but we are obliged to recognize in each a modification of equal and independent status. There are no systematic affinities among modifications (nor between the modes of different modifications).

Perhaps the most straightforward question that might be raised regarding these new ideas is why, given that it is posed a single organizational basis underlies all modification, there should be so much observable variety in our surroundings. This question is best answered by reading the rest of this work; for the present, it can simply be noted that we have no reason to expect that the rules of representation underlying the attribute spatial extension should be *directly* expressed in the sensible characteristics of the spatial domain. (Consider the analogous idea that while a particular integer has no direct expression in sensible space, it can still be used as a logically consistent means of labelling a certain quantity of entirely different objects.)

To recap our conceptual revision of the basic Spinozian model, the above comments have been diagrammatically represented in Figures 2 and 3. In both figures, attributes are assumed to bear the same general relation to substance described through Figure 1. Specifying (that is, both describing and exhausting its domain) M_1 in Figure 2 is the hierarchical class H_1 of finite modes m_i. Lines interconnecting modes 1 through 3 symbolize the relational system uniting them into H_1 with respect to A. The boundary of M_1 is dotted where it is not a part of a finite mode within H_1 because M_1 does not exist as sensible space where it is not expressed through H_1 (actually, H_i in general). Again, this extension of the Rationalist view seems to do no great violence to Spinoza's original way of casting things, but at the same time responds to the criticisms of his system made by Leibniz. An advantage of this conceptualization, moreover, is that it is particularly conducive to canonical hierarchical representation; i.e., a particular hierarchical class structure can be collapsed into a hierarchical class specified by a more inclusive set of internal relations. As indicated in Figure 3, the system of primary modes of Figure 2 might be combined in a fashion yielding a system of "second order" modes, a construction subject to the condition that the newly devised level of order be self-specifying at both its own level and the original one. This calls for a strategy of modal grouping that retains such relationships, a subject taken up shortly.

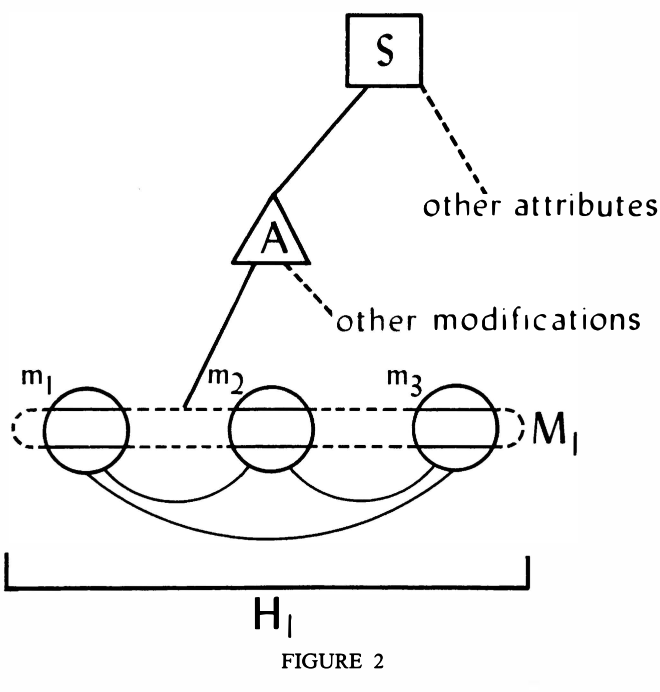

FIGURE 2

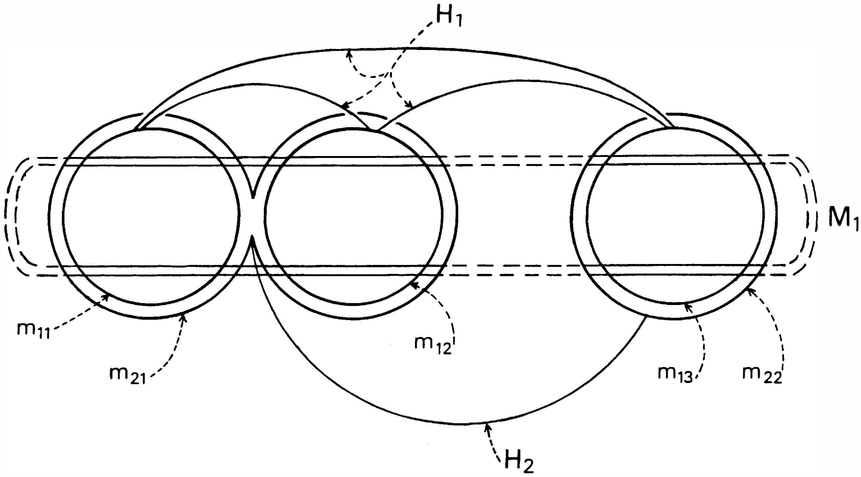

FIGURE 3

THE EMPIRICIST TRADITION VERSUS RELATIONAL ANALYSIS

At this point, it seems advisable to stand back for a moment to reflect on where we have been thus far and where we are going next. Faithful to the Empiricist tradition, it has become the norm to believe that the objects we choose to study exhibit attributes (in the more common sense of the word) that can be measured and reflected upon to give us direct knowledge of "underlying" reality. Thus, we now tend to believe it meaningful to consider "occupying space" and exhibiting measurable qualities to define space directly on the basis of those qualities. Following Spinoza, I do not believe this approach can produce an efficient representation of spatial structure and process. Order, I submit, is not most fundamentally manifest at the sensible level as a geometry involving objects; rather, it is a characteristic of the internal organization of the unobservable (namely, attributes—and note that I do not disallow the possibility of using geometry as an aid in understanding these). An attribute may be thought of as a particular kind of relational structure whose reflection in observable natural form is best represented by the notion of the hierarchical class. Should we wish to apply a theoretical understanding of the rules of representation within attributes to the sphere of the practical, therefore, we must first determine in what manner modifications internally differentiate into hierarchical systems of functionally interdependent modes.

A few examples of the general kind of thinking I am suggesting might be beneficial here. We can first return to the zoogeographic realm. Regional faunas develop as complex interplays between regimes of isolation (during which unique evolutionary lines develop) and regimes of physical connectedness (during which interchange of populations takes place). At any given time, the cumulative results of such processes are manifest in sensible space as an overlapping array of distributional ranges of populations. Individual populations within this array may be restricted to a particular region or found in several. On the basis of the distribution of such commonalities (which may be represented in matrix form as an array of similarities coefficients), the characteristics of each region can be stated in a way that in effect specifies a set of relations. Given the right regionalization scheme, this set of relations can in theory be expected to reflect the rules of representation inherent in the attribute spatial extension—being ultimately the link to their expression within a spatial domain (in this example, the terrestrial surface of the earth)—and thus constitute a hierarchical class. Similarly, the development of an "organism" is predicated on, literally, organization: the manner in which organal systems develop in a functionally interrelated way. In so doing, they constitute an internal ordering of relations that operates as a three-dimensional entity—a body—under the attribute of spatial extension. A more complicated example is provided by the evolution of central place hierarchies, in which the hierarchical classes emerging appear to be composed of modes (classes of central places) arranged in sensible space as a complex, intermeshing pattern of points rather than a single set of contiguous areal units. Nonetheless, the problem of study must be fundamentally the same, starting with the identification of the system of representation underlying the spatial distribution of central places, and only then proceeding on to substantive analysis.

If we wish to do more with this basic scheme of natural order, our attention must therefore be drawn to: (1) identifying the rules that order modifications internally in such a fashion as to constitute hierarchical classes; and (2) showing how the fact of such ordering can be interpreted within the real world of measurable qualities. Meeting these goals involves setting out logical criteria for the ordering of subclasses (modes) into hierarchical classes, and then translating this understanding into a framework within which metric (sensible space) interpretations are possible. I have not yet addressed the matter of the actual nature of modal differentiation; i.e., what specifically are the relational rules under which a hierarchical class organizes? Before we can attend to real world problems, it is necessary to start with an idea of what we believe should be the characteristics of relational space. The study of mammalian regionalization patterns, for example, therefore begins with a concern about the meaning of "varies over space" rather than any characteristics of the specific groups that are varying, including their distribution in sensible space. Ultimately, this concern should lead us to a particular delineation of regions upon which to base applied understandings of the modification "mammalian distribution." In related applications of such a system I would therefore be making use of an implicit relational structure to shed light on the manifest properties of the distribution of mammals, which is explicit within its spatial domain.

We now turn to a consideration of the meaning of "appropriate" systems of representation; i.e., ones that can be argued to mirror the internal relational structure of the attribute spatial extension.

THE DIFFERENTIATION OF SPACE INTO VERTICAL AND HORIZONTAL COMPONENTS

Randomness is, by common definition, lack of measurable order. Were there no order to our world, we would be unable to attach much meaning to any of it, or to meaningfully contrast any part of it with any other part. The fact that we can perform such contrasts, and obtain agreement about our pronouncements from other thinking beings, is proof enough that order exists. Another version of this remark involves pointing out that there are recognizable differences among things, and that these persist (in the mind, if nowhere else). These differences are relative; that is, some are greater than others. Beyond degree of difference among things, however, there is an even more important aspect of the character of natural variation. Some reflection suggests that differences organize themselves through independent structural axes that inherently generate a "horizontal/vertical" natural duality. Horizontal differences are differences that can be expressed in terms of distances, where "distance" is defined very liberally as a standard of comparison applied within a (any) metric. For example, most would agree that the "distance" between orange and red is less than that between orange and blue. Similarly, the relational "distance" (equals "similarity") between the faunas of northern North America and northern Eurasia is less than that between, say, New Guinea and Borneo, regardless of the actual miles involved in the geographic separation associated with these pairings. Horizontal differences are thus relational differences; i.e., their magnitudes can be ordered on the basis of established ordinal or interval scales.

"Vertical" differences, on the other hand, may be interpreted as hierarchical differences: the contrasts between a level of order and another it subsumes, or between itself and a level subsuming it.[5] Minerals and rocks form a relation of this type, as do organic molecules and organisms, and neighborhoods and cities. Vertical differences may thus be used to interpret patterns of inclusion; e.g., the vertical "distance" between cells and organisms is arguably greater than is that between cells and organs. The criteria for assessment here are non-relational in that they are based on impressions of degree of *functional* immediacy rather than on metric-based comparisons.

We may further refine the concepts of horizontal and vertical differences by referring to their expression as sensible space. With respect to horizontal differences, it is apparent that these must refer to entities that: (1) occupy different spatial domains, and (2) are logically equivalent elements of some greater whole. These constraints are necessary to the construction of logical comparisons (it is, for example, impossible to state a very meaningful difference between "lung" and "organism") and the metric associations made possible thereby. Vertical differences, by contrast, refer to entities that: (1) (at the least) share portions of the same spatial domain, and (2) function in large part independently of one another (as in the hierarchy consisting of "atom", "cell", "organism", "community", etc.).

This appraisal of the character of sensible space can be used to link what we observe as sensible space to the Neo-Rationalist views introduced earlier. Obviously, the two kinds of relational conditions just described are integrated, together yielding a single sensible space in which both areal and vertical differentiation and organization are involved. I should now like to consider a means of eliciting a relational space comprised of vertical and horizontal components; i.e., one in which hierarchical differentiation out of increasingly inclusive levels of organization is complemented by intra-level differentiation. The philosophical position developed earlier provides a conceptual framework for such interrelation; it does not, however, suggest particulars of organization (e.g., how many hierarchical levels there can be, what number of modes may be associated with which levels, etc.). These particulars are deduced here through an argument involving simple combinatorial statistics. This tactic allows us to translate the Neo-Rationalist argument into a form more appropriate for considering the nature of the rules of representation within the attribute spatial extension. Once we have proposed a particular model of these rules, it will be possible to consider how these might be expressed in/as sensible space, and ultimately to apply this knowledge to the study of real world distributions.

A MODEL OF HIERARCHICAL INTERNAL RELATIONS

The model of relational space about to be offered is predicated on Einstein's "lazy universe" principle: that universal change always follows the route of least resistance (thereby conserving energy). The physics of Einstein's principle, however, is inspired by an overly simplistic understanding of the relationship between vertical and horizontal differentiation properties. Reductionist physical models are not usually viewed as contradicting the basic hierarchical structure of nature (as represented, for example, by the Miller hierarchy[6]), yet such views are clearly incomplete for their lack of attention to the crucial problem of how it is that within-level structural uniquenesses can exist. Modern physics can make little sense of uniquenesses; physical models do not exist that are of much value to the fundamental interpretation of the organization of, for example, central place hierarchies, world faunal regionalization patterns, or organal systems (which is not to say that they cannot sometimes be used to provide (usually) analogical *descriptions* of these).

Yet it would be singularly useless to take the position that the unique structures that emerge under the operation of simple physical laws cannot after all be related back to those laws. A solution to this dilemma is suggested by the canonical structure advanced earlier. It will be recalled that the Spinozian attribute spatial extension has been portrayed as a system of representation operating under a single set of relational rules. Figure 3 schematized the canonical nature of this model of spatial extension; the substructure of particular modifications can be viewed through hierarchical classes involving various numbers of modes. We should be aware, however, that a modification comprised of but one mode can provide us with no information regarding the relational structure of the underlying attribute: it itself exhibits no such structure. Technically, therefore, we cannot recognize a hierarchical class in a unimodal modification. (The same reasoning holds for the bimodal case; see later discussion.) This is not to say, however, that such a structure can have no measurable characteristics at all. Indeed, those characteristics existing "in" (and in another sense, "as") all unimodal entities might be understood to represent the commonalities upon which physics is based. As modification-level phenomena, these commonalities can also be associated with plurimodal entities; all this says, as we should expect, is that all levels of organization can be reduced to certain *kinds* of fundamental properties (which nonetheless cannot serve to describe their associated manner of function *in entirety*[7]). One of the most fundamental of these is the tendency for a system to increase its entropy level—that is, to move towards its most-probable-state. I see no more conservative starting position for a consideration of modal differentiation characteristics than to assume that the same general principle holds in the development of the plurimodal structures identified in hierarchical classes. Thus, we might speak of the relational structure underlying spatial extension as being one out of which "unlikely" states of matter are produced in the most likely way. This last concept is not by any means novel; for a discussion of the relevant physics, the reader is referred to the literature on irreversible thermodynamics.[8]

I am thus taking as given here that the combined verticality/horizontality of nature has fallen out in a manner referable to a most-probable-state kind of process. Our task therefore becomes to suggest a hierarchical structure of relations whose component sets of modes (hierarchical classes) describe such a state. (Note that we are confining our attention to particular modifications, and not the sum of all modifications.)

We can begin our consideration of this kind of hierarchical ordering by asking the simple question "Given n objects about which we know nothing, what is the most likely number of classes into which these n objects can be grouped?" This elementary problem in combinatorial mathematics may be symbolized as $\max\binom{n}{r}$ and is solved by finding the maximum value of S given by:

$$S = r! \Big/ \prod_{i=1}^{a} r_i!$$

We can enliven the discussion a bit by giving it a more relevant context. Given that n equals "number of subclasses" and r equals "number of classes into which n may be grouped", what values of n and r may be associated with one another such that these will yield maximum values of S for a given n? The most famous example of this type of problem is in statistical mechanics, where n is the total number of units of energy available to an isolated system and r, the number of elementary particles absorbing those units. It is assumed that these particles can attain various levels of excitation within the system; the total energy of the system is thus absorbed through certain numbers of particles attaining certain excitation levels. For any given combination of n and r, most-probable-state solutions for the number of particles falling into each excitation level can be worked out (this distribution is approximated by Boltzmann's Law). It is almost possible to translate this example directly into terms appropriate to the discussion here, as "number of units of energy available" becomes "number of subclasses", "number of elementary particles in the system" becomes "number of classes", and "excitation level" becomes "number of subclasses in a given class". One important difference between the two situations exists, however: our relational structure can include no analog to the "ground state" condition (excitation level zero) in statistical mechanics, as it is clearly illogical to permit a class containing zero subclasses to exist within an inclusive hierarchical structure. Thus, the distribution of classes that can be grouped from some initial number of subclasses is of truncated nature.

A short example is perhaps useful. Under the conditions stated, it turns out that seven subclasses can be grouped into four classes in more ways (equals more probably) than seven subclasses can be grouped into any other number of classes. This is detailed as:

	No. of classes		No. of subclasses in a class		No. of subclasses
	1	×	3	=	3
	1	×	2	=	2
	2	×	1	=	2
totals:	4 classes				7 subclasses

This configuration of one class with three subclasses, one class with two subclasses, and two classes with one subclass, may be arrived at in 4!/1!1!2!, or 12, ways. Recalling Figure 3 in Part I, however, we need not stop at this first ordering of subclasses. Our four classes can be further grouped into some most probable number of still more inclusive classes. This turns out to be three, for which the details are:

No. of classes		No. of subclasses in a class		No. of subclasses
1	×	2	=	2
2	×	1	=	2
totals: 3 classes				4 subclasses

This configuration can be arrived at in 3!/1!2!, or 3, ways.

It is apparent that a ladder of subclass-class relationships may thus be generated in which any initial number of subclasses is canonically grouped through x steps to one all-inclusive class. Below are listed all the class inclusion series for initial values of one through ten meeting the maximum likelihood conditions just set out. Note that some outcomes are equiprobable, necessitating the expanded listing:

Initial number of subclasses → Increasing inclusiveness → Unity (1)

1
2.....1
3.....2.....1
4.....3.....2.....1
5.....4.....3.....2.....1
6.....3.....2.....1
6.....4.....3.....2.....1
7.....4.....3.....2.....1
8.....5.....4.....3.....2.....1
9.....5.....4.....3.....2.....1
9.....6.....3.....2.....1
9.....6.....4.....3.....2.....1
10.....6.....3.....2.....1
10.....6.....4.....3.....2.....1

None of this would be especially intriguing were it not for the fact that it is possible to isolate structural solutions for the above groupings that maintain most-probable-state characteristics across *all* levels of the grouping structure. This can most easily be appreciated through the aid of dendrograms of the hierarchical relationships involved. A possible dendrogram representation of the seven subclass-four class configuration discussed above is:

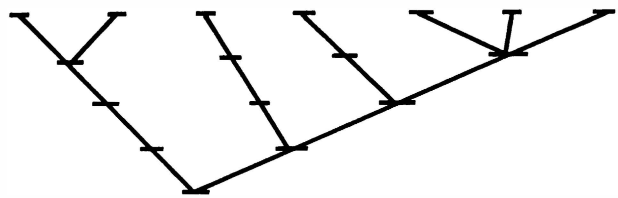

FIGURE 4

This shows how seven "first-order" (sub)classes group most probably into one second-order class containing three first-order (sub)classes, one containing two sub(classes), and two containing one (sub)class. But it also suggests that these seven first-order classes group into three third-order classes, two fourth-order classes, and one fifth-order class. Moreover, implicit in the diagram is the idea that the four second-order classes group into three third-order classes, two fourth-order classes, and one fifth-order class. Further, the three third-order classes group into two fourth-order classes and one fifth-order class and the two fourth-order classes group into one fifth-order class. Diagrammatically, the inclusion series may be represented on a "by-order" basis as:

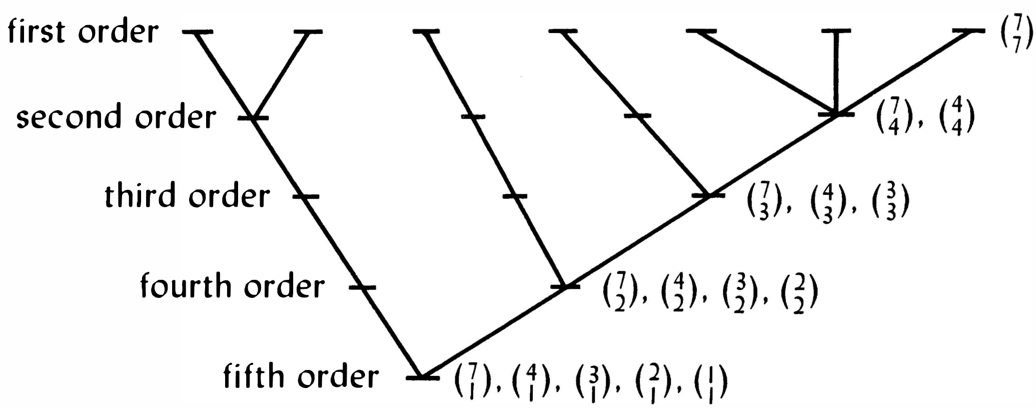

FIGURE 5

Now it so turns out that each of the orderings represented in the particular "7-4-3-2-1" series depicted above is the most probable such ordering (for example, the ordering $\binom{4}{2}$ in the dendrogram above produces fourth-order classes containing three and one subclass, respectively, and this is what is required under maximum-likelihood conditions). This is therefore what might be referred to as a "maximum-likelihood tree". That such a relational structure exists is significant for our overall discussion, because: (1) it provides a reasonable interpretation of the system of canonical representation demanded by our earlier-developed concept of the hierarchical class, and (2) it satisfies our hope that a combined structure of horizontal and vertical relations can be stated in most-probable-state terms.

There are a relatively small number of trees that satisfy this most-probable-state

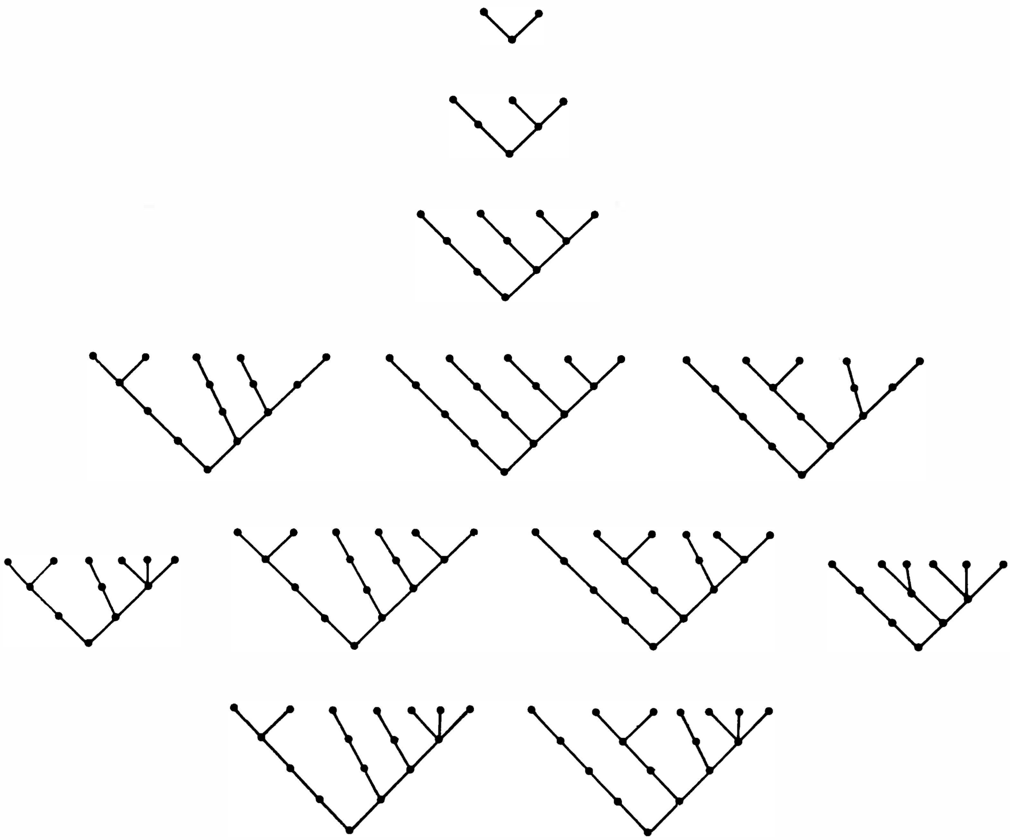

FIGURE 6

canonical hierarchic inclusion criterion. The solutions for initial (sub)class values of up through seven are given in Figure 6.

A feature of this canonical inclusion series of considerable interest is the way that two initial subclasses (or a more extended series that has been collapsed to the stage of two subclasses) behave as collapsed a final time. It turns out that two such subclasses group equiprobably into one class of two subclasses or two classes of one subclass each. The tree structures involved, however, are of course identical. This is curious, because it seems to indicate that we must interpret what for all apparent purposes are identical structural outcomes as being logically different from one another. The solution to this dilemma seems to be to recognize in these alternative class structures another aspect of the hierarchical nature of unity itself. Given only two subclasses at the outset, there can be no way to compare one to anything else in a way which is different from an analogous comparison to the other.[9] As a result, we are obliged to view a pair of subclasses as being both complementary images of one another and a single whole. Left and right, East and West, negative and positive electrical charges, the complementary helical strands of the DNA molecule, and male and female provide some real world examples of this condition. There are important philosophical ramifications of this interpretation. Since any initial number of subclasses can ultimately be hierarchically grouped to unity, this means that any degree of initial complexity of elements can be reduced to an interpretation based in structural isomorphisms. This will be true, moreover, wherever a bifurcation in the tree of structure exists—that is to say, *everywhere* (the apparent special case of "bushes" presents no problems for this interpretation; see below). Spinoza's and Leibniz's basic view of the unity of nature is thus supported in another way, because it can be seen that it is impossible in such a universe to have a spatially-extended entity that is other than a reflection of another such entity, in association with which it constitutes a logical whole.

(As an aside, I feel I should note that the comments immediately above would seem to have considerable relevance to the theory of knowledge. In particular, they suggest a way of understanding the progression of "thesis-antithesis-synthesis" that is invariably used to characterize scientific study (and other social and natural processes). I am now preparing a short work that will feature a discussion on some related thoughts.)

To this point, our dendrogram representations of most-probable-state hierarchical relations have been constrained in a way that is probably overly restrictive. For example, in Figure 5 it can be seen that all first-order subclasses that group together at the second-order level of integration do so at the same distance up the tree. On the basis of ideas set out so far, I see no reason why this additional requirement need be met in a maximum-likelihood tree. Bushes such as that evident in the first-order level of the tree in Figure 5 may thus contain latent structure *within* a given hierarchical level; to maintain the whole system of relations, however, it is necessary to ignore such latent structure and understand each subclass involved as being logically equivalent.

With this refinement, we are in a position to identify the complete set of maximum-likelihood trees that exist. These are portrayed in Figure 7. Eleven appears to be the highest number of initial subclasses that can be integrated into a maximum-likelihood tree. Arrows within the diagram indicate the "lineage" of derivations associated with increasing complexity.

Figure 7 shows that it is possible to construct a reasonably complex and internally-logical hierarchical system of class-subclass relations out of a bare minimum of initial constraints. Note, moreover, that even the limitation of eleven initial elements in the system is not much of a constraint, since any degree of further elaboration of structure *within* each initial subclass is non-contradicting as long as: (1) such elaboration is recognized as being irrelevant to the logical status of that subclass, and (2) it is understood that such elaboration can contribute to no further hierarchical ordering within the system.

The availability of this system gives additional substance to the philosophical discussion presented earlier, since we are now provided with a possible means through which to understand canonical hierarchical representation in sensible space. But we still lack a means of applying the overall model to the study of the distribution of things in sensible space, as we have not yet considered how the representation system we have posed might be constrained when expressed *as* a sensible space of orthogonal dimensions. This is a key issue. Our immediate goal, it should be remembered, is to describe the system of representation underlying the attribute spatial extension. It is thus critical to show that the system we are defending can be expressed *as* the sensible space to which we can apply measurement.

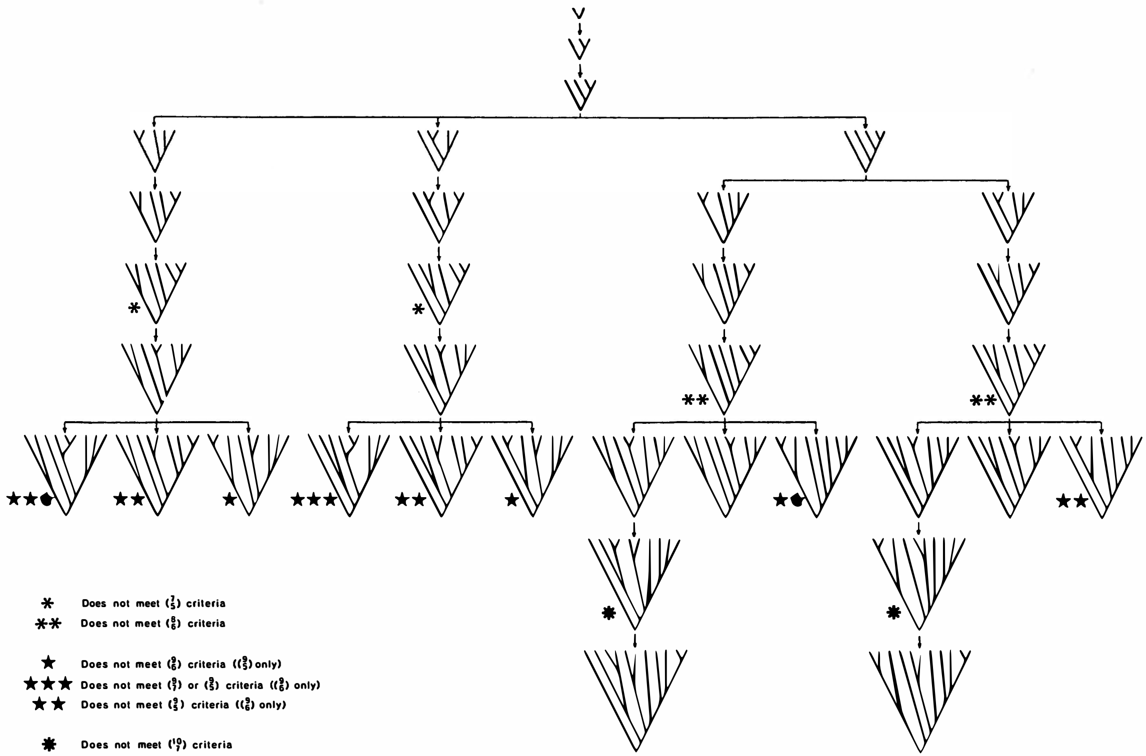

FIGURE 7 Dendrogram representations of all maximum-likelihood trees as defined in the text. Arrows indicate "lineages" of derivation. A number of the level nine trees do not produce maximum likelihood class structures for both max$\binom{9}{5}$ and max$\binom{9}{6}$ conditions; these are especially noted. In addition, it is noted that several of the trees represented are unable to produce maximum likelihood class structures for certain other combinations, but as the latter do not directly contribute to the hierarchical inclusion series as here defined, the importance of this is debatable.

REDUCING SENSIBLE SPACE TO ITS UNDERLYING RELATIONAL BASIS

In speaking of the "entropy" of a system, we are usually referring to some measure of its level of disorder. This has sometimes given rise to the impression that a progression toward increasing order, or negentropy, represents simply the opposite of a trend toward disorder. This is not the case. Minimizing entropy and maximizing negentropy are not equivalent concepts, as is suggested by the fact that there is no single or absolute kind of order. There are perhaps an infinite number of ways of "maximally" ordering things, but, in effect, only one kind of maximum disorder: randomness. While it is not difficult to subject a particular pattern to test against the hypothesis that it is "unordered" (e.g., was generated by a Poisson process), this activity in no way provides a general context within which particular instances of non-randomness might be related to one another. Even tests that distinguish clustered versus regularly-spaced patterns are at their heart measures of deviation from the random (equals most probable) state, and do not resolve the question of what order *is* (rather, they expose particular *kinds* of order on the basis of various assumptions about what order *is not*).

According to present views, when we attempt direct analyses of pattern, we are setting up hypotheses about the characteristics of sensible space only. Earlier I supported the Rationalist view that a single underlying relational space provides the basis of order for sensible space. Now we will be interested in the idea that, given an appropriate understanding of order at the relational space level, it might be possible to derive methods of analysis exposing order at the sensible space level, *regardless of the perceived form the latter takes to express the former*. In effect, therefore, I will be defending the proposition that the patterns manifest in sensible space—despite their infinite variety—are yet the product of a single, portrayable organization process.

Adding together comments made to this point, I submit that it may be possible to interpret sensible finite structure as the physical manifestation of a canonical ordering of relational (class/subclass) states according to maximum-likelihood criteria such as those set out earlier. While we cannot study the attribute spatial extension directly as if it were somehow a sensible characteristic of its own modifications, we might study the way in which it manifests itself through the internal ordering of those modifications. In short, we can perhaps confirm a particular model of the internal structure of attributes by examining the way that finite structures (modes) are reflected in one another.

This we have so far dwelled on using an abstract maximum-likelihood-based perspective. Again, however, it cannot be supposed that merely because we have provided a possible way of comprehending complexification (to borrow Teilhard de Chardin's term) that we have also provided a useful means of relating this knowledge to the measurable characteristics of sensible space. This must be accomplished through independent means, but in such a fashion as not to conflict with the maximum-likelihood interpretation.

A negentropy-accumulating system is one which virtually by definition has developed in such a way as to increment the differences among its component parts. We saw earlier that the degree of difference between two things might be stated as a distance between them. For three items, the system of differences involved becomes in this manner a system of three interrelated distances; for example, among locations set within an n-dimensional space. For the latter kind of system of distances to represent negentropy-maximized relations, it must have the property that the distance from each point to all other points be the same, or at least some approximation of this (since the combination of number of points and designated dimensionality of the space will constrain the perfection of self-representation of the solution). The resulting cloud of uniformly packed points may be interpreted as depicting an entropy-minimized condition for the simple reason that it is variance-minimizing with respect to the sum of distances squared among the elements of the system. But it can also be interpreted to reflect a negentropy-maximized condition, as the relational representation involved may be of a system whose subelements are relatively as unique as they can be (i.e., whose relative dissimilarities with respect to their measurable characters are as great as they collectively can be).

The significance of this manner of representation is that it is both explicitly relational and provides the possibility of an interpretation of highly negentropic structures, *regardless of their specific characteristics of occupation of sensible space.* Suppose, for example, that we have generated a set of zoogeographic regions such that each is as unique as it can be; that is, such that the standardized sum of differences among all have been maximized. Thus, the characteristics of a modification—faunal distribution—have been partitioned into a number of finite modes; these inherently depict a system of representation if for no other reason than that the conditions of faunal element inclusion in each region directly influenced decisions regarding the identification of the areal domain of each region (and *vice versa*). The perfection of this reflection of the identity of each region in each other region can be assessed in the degree to which an *n*-dimensional mapping of the relational structure of the system approaches even packing. With optimum classification results, therefore, we might obtain a system of relations which reflects both the diversity of real world conditions *and* the fact that each unit looms as an equal contributor to the representation of that diversity.[10] I have discussed some related problems in more detail elsewhere[11] and am in the process of preparing further treatments.

The conditions of relation within a system such as the faunal regions one can be stated as a matrix of similarities. Two common means of gleaning information from a similarities matrix are hierarchical cluster analysis ("classification") and multidimensional scaling ("MDS"). Through the first, a dendrogram similar to the ones discussed earlier may be generated that expresses the hierarchy of relationship (similarity) among classified units. Through the second, an *n*-dimensional map expressing similarities as a set of distances (anchored by locations) in an *n*-dimensional Euclidean space is produced. This particular plurality of form of system representation suggests an interesting question: can the maximum-likelihood trees we constructed simultaneously be expressed as logically-consistent minimum entropy *n*-dimensional maps? That is, can the relationships underlying the canonical hierarchical class structure derived earlier also be expressed *as* a sensible space of orthogonal dimensions? This should be so, if sensible space can indeed be interpreted as the physical manifestation of the underlying relationships postulated. Being so, we would have at hand the beginnings of a method whereby any hierarchical class might equally well be expressed as a maximum-likelihood relational tree or an entropy-minimizing structure operating as extended space under a certain number of degrees of freedom. We could thereby conceptually transform any delimitable system (modification) into its equivalent basis of canonical internal representation. This would provide a base through which practical studies of hierarchical class organization might be initiated.

I have recently completed an analysis designed to shed some light on the feasibility of this approach. Before discussing this work, however, it is necessary to introduce some relevant comments about equilibrium and disequilibrium systems.

THE SPATIAL REPRESENTATION OF EQUILIBRIUM AND DISEQUILIBRIUM SYSTEMS

Were extended space unchanging, it would be possible to characterize all systems existing within it as being in a state of static equilibrium. This, of course, is not the case, and as a result we have invented terms to describe other kinds of relations between systems and their environment. Where a system is structurally unchanging and its various inputs and outputs are stable and balanced, we describe it as being in "steady state" equilibrium. Where a system's internal structure changes but its inputs and outputs remain balanced through that change, we recognize a "dynamic equilibrium." A third state, disequilibrium, is produced when a system's internal structure changes in relation to input/output imbalances.

Unfortunately, it is extremely difficult to unambiguously apply these descriptors to real world conditions. The main reason for this is that the above categories are not, for practical reasons, mutually exclusive. A good example of the problem can be seen in the way the global energy balance is usually related to evolution. In terms of energy throughput and transformation, the earth's surface defines very nearly an equilibrium steady state system. In view of this constancy of input and output, surface physical and biological evolution processes are sometimes interpreted as exhibiting a kind of dynamic equilibrium, but it seems that this intepretation should be reserved for systems that are changing, but whose total negentropy is remaining constant (at least in the relational sense—see later discussion). This does not appear to characterize the present (or past) surface of the earth, which would therefore be better understood as a system in disequilibrium.

These matters are relevant to the present discussion because it is not initially obvious whether the most-probable-state structural formulations developed here can describe modifications under all kinds of equilibrium conditions. Clearly, any natural system undergoes growth and change, and the concept of the modification would be unrealistically inflexible if it did not take this into account.

We need not dwell on the static and steady state equilibrium states, as neither has much relevance to the present context. No system that changes can be associated with the first condition, and the second cannot really be understood here as distinct from dynamic equilibrium (as will become apparent shortly). We are therefore left with the problem of how to use the approach developed here to describe negentropy-maximizing systems under conditions of either disequilibrium or dynamic equilibrium.

Characterization of the latter state through present ideas is fairly straightforward. We must first recall the Euclidean space representation of relations within a negentropy-maximized, *n*-mode modification. As described earlier, this is a "cloud" of maximally-spaced points. Under strict steady state dynamic equilibrium conditions, a system thus represented in mimimum entropy terms may still change (that is, its particular manifest characteristics of uniqueness may), but only if that change does not result in a disturbance of intra-modification relations leading to an increase in entropy within the system (which would be evident in the Euclidean space representation as reduced "packing" within the cloud of points). It is therefore possible to imagine an evolutionary process initially characterized by disequilibrium continuing under dynamic equilibrium steady state conditions. Real world illustrations can easily be imagined. Conditions of relation within a maximally-differentiated system of faunal regions will remain unchanged, for example, if within each region a different form goes extinct at the same time (or evolves anew, or if both happen, etc.). Post-maturation aging processes in individual organisms might also be interpreted along these lines.

The key problem in trying to understand disequilibrium conditions on the basis of the most-probable-state organizational structure advanced here concerns how to deal with the fact that a system can remain thermodynamically stable even as it changes over time. In a growing animal, for example, we see a good case of a system undergoing irreversible change, but all the while maintaining (in reality, approaching) steady state relations with its environment. The earth's surface system as described earlier provides another example. Through the present model we can interpret these as modifications whose component modes are undergoing a constrained kind of relational adjustment. For historical reasons, newly-formed modifications are likely to exhibit degrees of inefficiency of inter-modal function; i.e., some modes will be contributing less unique input to overall system operation than others. Subject to the constraint that the changes involved will not interrupt the *continuity* of intra-modal information flow, further modal differentiation will take place as feedback-initiated processes reduce functional overlap. Simple thermodynamic integrity can thus be regarded as maintained as long as each mode continues to contribute information to the operation of the overall system in amounts sustaining the demands of pre-existing structure. (The proximate cause of organismal death, for example, is usually the collapse of some vital subsystem.) Irreversible system change occurs, meanwhile, as modal structures evolve relative to one another in such a way as to gradually minimize overall redundancy of internal function.[12]

It is thus argued that the present understanding can be used to deal conceptually with modifications characterized by either equilibrium or disequilibrium. With this knowledge, we can turn to discussion of a simulation designed to investigate whether the aspatial, most-probable-state-based hierarchy determined earlier might be expressed as a three-dimensional sensible space.

A SIMULATION OF A MOST-PROBABLE-STATE THREE DIMENSIONAL SPACE

As has been explained, it is useful to arguments presented here to determine whether the particular relational structure deduced earlier might also be interpreted as a space of Euclidean dimensions. One way of looking at this matter involves use of multidimensional scaling as a simulation device. The MDS package KYST-2A[13] was first used to create minimum entropy representations of relational systems comprised of varying numbers of elements. This was accomplished by reading matrices of "1's" into the metric version of the program and instructing it to produce three dimensional configuration solutions of high precision. Output configurations were thus forced to consist of maximally packed arrays of point locations. The symmetric distance matrices derived from these configurations were then used as the input for cluster analyses based on the information statistic approach.[14] It was hoped that the resulting classifications would consist of elements canonically organized into class structures mirroring those identified in Figure 7. Positive results would support the idea that the relational structure postulated here might in fact be capable of representation as a three-dimensional Euclidean space.

Systems comprised of from three to eleven elements were examined. (Larger numbers of initial elements were not considered because results reported earlier indicated that no more than eleven initial units could be integrated into a maximum-likelihood tree, regardless of other considerations.) Only systems comprised of three, four, and five elements could successfully reconstruct a full maximum-likelihood tree from the intraconfiguration distances. Nine element systems also produced maximum-likelihood structures at each individual hierarchical level, but these proved incompatible with one another (i.e., the solutions could not be integrated into a single inclusion series).

It might be asked at this point what this exercise proves, given that most or all real world systems are not in the equilibrium state tacitly assumed in this simulation. In short, why not base the attempt to simulate a space of Euclidean dimension on the relations inherent in an intraconfiguration distance matrix describing any given *nonequilibrium* system? This criticism is difficult to overcome unless we accept a kind of separation between geometry and function in the present model much like that now taken for granted in physicalist understandings of universal structure. Through relativity theory and its employ of non-Euclidean geometry as a modelling device, for example, we are able to resolve certain physical difficulties (e.g., the predictable effect gravitational fields have in "bending" light) occasioned by our prior acceptance of a three dimensional Euclidean perspective. This does not mean, however, that space itself need be considered in some sense "curved."[15] Here, it becomes necessary to postulate something very similar regarding the ability of disequilibrial systems to sustain a Euclidean sensible space (roughly, the greater the level of disequilibrium, the greater the expressional "distortion" involved in "stretching" it into Euclidean form). Interestingly, the difficulty seems to disappear entirely if we understand the universe as a whole to be a closed, steady state dynamic equilibrium system characterized by negentropy, as well as entropy, conservation: through present views the representation of such a system is as in the simulation just performed. I shall develop this theme further at another time.

In any case, I conclude from the simulation results just presented that a maximum-likelihood sensible space of three Euclidean dimensions can be based on a system of relations characterized by hierarchical classes involving *at least* five modes. These findings cannot be considered the last word in such investigations, of course. The relational structure I have treated here can doubtlessly be scrutinized in other ways, and arguments favoring the consideration of entirely different structures cannot be rejected *a priori*. Regardless, there now seems to be at least some reason to proceed on the assumption that a relational space of the type imagined is also a physical possibility.

DISCUSSION

In view of arguments set out thus far, it is possible to continue onward to the matter of the study of real world systems. I do not wish to give the impression, however, that I feel I have in any sense "proven" (or even attempted to prove) the relationships I have advanced; rather, I intend to treat the preceding discussion as a model that can promote hypotheses and be subjected to test in various ways. (Note, of course, that such tests will involve particular understandings of the internal relational structure of the attribute spatial extension, and not the philosophical position that some such structure of this kind exists.)

In Part II of this work, two initial examples of application of the Neo-Rationalist approach espoused here will be discussed. Before these can be presented, however, it is necessary to consider some technical matters relevant to the framing of testable hypotheses within the current model.

I have so far been persistent in claiming that space (or, more exactly, the attribute "spatial extension") consists of a structured relational framework. This notion remains comfortable, however, as an abstraction only. We do not directly perceive "relational frameworks"; rather, we are more likely to infer that such structures exist on the basis of reasoned appraisals of perceived similarities. In the identification of similarities among things there is secondarily the recognition that similar processes were responsible for creating those things. Here, we are concerned with the idea that a *single* process might be identified that can explain the characteristics of differentiation *within* any spatially-extended entity (i.e., modification). We have already seen how the differentiation process might produce unique substructures (i.e., modes) and still organize itself around a flow of information better viewed as a system-level phenomenon. System description may feature either persepective, but in practice we usually find it easier to concentrate on structure first. As a result, we often find ourselves trying to determine the mode of information processing within the system from some set of data representing the similarities among its substructures. Following present ideas, we may imagine such processing to have two general components relatable to similarities. There is first the matter of "absolute" similarity. It is straightforward that absolute degree of dissimilarity among modes provides a direct indication of information content of the system in sum. Simply, as total uniqueness "per mode" increases, so too, we may assume, does the total information content of a given modification. Whether relatively simple or relatively complex, however, a given system can be expected to modally differentiate in such a manner as to continue to minimize the *relative* similarities among its parts. It has already been posed here that this reduction of redundancy in internal structure—a disequilibrium process—can only take place subject to the system's maintenance of internal equilibrium conditions, this being most easily recognized as a continuing balance of flow of information among the modes within it.[16]

The matter of the interplay of "relative" and "absolute" similarities among modes becomes important when we attempt real world descriptions leading to the identification of hierarchical classes. In the simulated systems investigated earlier, the matrices of Euclidean distances forming the basis of the clustering operation could be considered direct mappings of within-system information flow under steady state, dynamic equilibrium conditions: given the absence of weighted elements, all that can be said about the system is summed up in the distance relations. A matrix of similarities describing a real world modal system, however, is not likely to provide such a direct representation of the information flow involved. Though the latter might be understood as being implicit in the former, we must remember that similarities are usually evaluations of differences between measured *structures*, and not of the "actual" flows between same. Additional steps are therefore needed under these circumstances to determine whether a hierarchical class structure is at hand.

A simple way of ascertaining whether a particular matrix representation of intrasystem affinities indeed describes a hierarchical class involves an iterative procedure known as "double-standardization."[17] When a matrix is double-standardized, its row and column elements are alternatively re-calculated into Z scores as many times as it takes to make the latter converge to stable values. This operation implicitly "de-weights" the elements of the matrix; for example, the procedure is often used as a means of compensating for size in the study of matrices of commodity flows between places of different populations. In so doing it provides an entropy-maximized restatement of intra-system relations.[18] While any symmetric matrix can be double-standardized, we should expect the double-standardization of a similarities matrix portraying a hierarchical class structure to yield very particular results. Specifically, the array of standardized values produced should be symmetric (i.e., i, j elements will equal j, i elements). We have already supposed that hierarchical classes characterize modal structures through which move balanced flows of information. As double-standardizing inherently "de-weights" the initial elements, we should therefore expect to find transformed values directly mirroring this kind of exchange.[19]

As double-standardization should in theory expose any hierarchical class structure, it becomes the first step in exploring the nature of the system (modification) at hand. The second step involves determining the system's internal redundancy characteristics—i.e., whether it is equilibrial or nonequilibrial. Degree of redundancy can be ascertained through simple correlation analysis of the similarities matrix. Where a system has reached steady state dynamic equilibrium, we interpret this through the present model to mean intra-system redundancies have been minimized; hence, modal relations will be maximally uncorrelated. Degree of disequilibrium will thus be evident in the extent to which the mean correlation value for a correlation matrix derived from a given similarities matrix deviates from the lowest possible such value. For a five modal system, the latter value is approximately $r = -0.250$ (averaged from the non-diagonal elements only).

It should be understood that in many or most instances, the exact limits of the spatial domain of each modal structure will not initially be obvious. More often than not it will be necessary to find some means—for example the information statistic clustering routine mentioned earlier—to determine "OSU's" (operational spatial units) in such a fashion as to expose the negentropy-maximizing structure inherent in a given modification. If this has been successfully accomplished, double-standardization will confirm this. If the results are not confirmed, it will mean one or more of the initial conditions for hierarchical class recognition have not been met, and appropriate adjustments will be necessary. The nature of, and reasons for, some such adjustments will be elucidated in the context of the analyses presented in Part II.

In the preceding paragraphs I have described (proposed) necessary and sufficient conditions for hierarchical class recognition in sensible space. Once the basic structure has been identified, various kinds of analysis involving testable hypotheses are possible. Examples of the latter, and their bearing on the matter of model validation, are discussed in Part II.

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9. M. D. Mesarovic and D. Macko, "Foundations for a scientific theory of hierarchical systems." In: *Hierarchical Structures*, edited by L. Whyte, A. G. Wilson, and D. Wilson, American Elsevier, New York, 1969, pp. 29–50. Note that this implies bimodally-differentiated modifications also provide no basis for the representation of the attribute spatial extension.
10. This description oversimplifies matters, as will become apparent shortly.
11. C. H. Smith, "A system of world mammal faunal regions. I. Logical and statistical derivation of the regions." *J. Biogeogr.*, **10,** No. 5, 1983, pp. 455–466; C. H. Smith, "A system of world mammal faunal regions. II. The distance decay effect upon interregional affinities." *J. Biogeogr.*, **10,** No. 6, 1983, pp. 467–482; C. H. Smith, "Areographic representation of faunal characteristics through a 'second order' relational approach." *Evol. Theory*, **6,** No. 5, 1983, pp. 225–232.
12. A good analog of this kind of system change toward higher order can be obtained by tracing an MDS operation involving a starting configuration of points all located at the origin (or randomly placed), and operating on an input similarities matrix comprised solely of "1's". With each passing step of the calculation algorithm, solutions converging towards a maximum spacing of points will be produced; the nature of this convergence closely parallels what I suppose to take place during the "maturation" of a negentropy-maximizing system (modification). In this context we should mention two relevant (and historically important) papers by M. Maruyama: "Morphogenesis and morphostasis." *Methados*, **12,** 1960, pp. 251–296; "The second cybernetics: Deviation-amplifying mutual causal processes." *Am. Scien.*, **51,** 1963, pp. 164–179.
13. J. B. Kruskal, F. W. Young, and J. B. Seery, "How to use KYST-2A, a very flexible program to do multidimensional scaling and unfolding." Bell Telephone Laboratories, 1977 (unpublished).
14. R. J. Johnston and R. K. Semple, *Classification using Information Statistics*. Concepts and Techniques in Modern Goegraphy, No. 37, Geo Books, Norwich, England, 1983.
15. I. W. Roxburgh, "Is space curved?" In: *The Encyclopaedia of Ignorance*, edited by R. Duncan and M. Weston-Smith, Pergamon Press, Oxford, 1977, pp. 85–89.
16. Through application of the "absolute" and "relative" similarities notions, an additional difficulty may be dealt with. A relational system that has reached the dynamic equilibrium state will always be characterized by minimum internal redundancy; this will indicate that the *relative* dissimilarity

among modes has been maximized. *Absolute* difference among modal structures, however, might well continue to increase (or decrease?) after the steady state has been reached (as in the faunal system example discussed earlier). Lacking this understanding, we would have difficulty explaining the mere fact that degree of absolute difference among modes varies from system to system.

17. For a related example of use, see: P. B. Slater, "Hierarchical internal migration regions of France." *IEEE Trans. Syst., Man, Cybern.*, April, 1976, pp. 321–324.
18. A. G. Wilson, *Entropy in Urban and Regional Planning*. Pion Press, London, 1970; P. B. Slater, "Hierarchical internal migration regions of France." *IEEE Trans. Syst., Man, Cybern.*, April, 1976, pp. 321–324; P. Gould, "Pedagogic review." *Ann. Assoc. Amer. Geogr.*, **62**, 1972, pp. 689–700.
19. The fact that an entropy maximization procedure is used here to expose a negentropy-maximizing structure is not coincidental. Entropy maximization (see references in note 18) implicitly converts an initial matrix of flows (or similarities) into a most-probable-state version of the same. In the present model order is conceived to evolve in the most probable fashion; we should accordingly expect to find the relations among modal structures expressed in this fashion. We can thus understand a system's entropy to continue to be maximized even as it reaches higher levels of order, a resolution conforming with the second law of thermodynamics.

Charles H. Smith was born September 30, 1950, in Winsted, Connecticut. After finishing secondary school he entered Wesleyan University, Middletown, Connecticut, and graduated with a B.A. in geology in 1972. His interest turning to geography, he took an M.A. degree in that subject from Indiana University, Bloomington, in 1979, and a Ph.D. from The University of Illinois, Champaign-Urbana, in 1984. His research reflects interests in the history of science, zoogeography, ecological evolution, classification, and General Systems Theory. The work presented here began as an inquiry into faunal classification methods; it became apparent, however, that the theoretical structure emerging suggested other kinds of applications as well. As resistance to the position developed here is anticipated, future treatments are likely to emphasize further deductions leading to the development of testable hypotheses.