Matched Pair \( t \) Procedure

When we have paired sample data \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), such as with “Before/After” measurements of the same person, then the samples \([x_1, x_2, \ldots, x_n]\) and \([y_1, y_2, \ldots, y_n]\) are not independent. Thus, two-sample procedures for the difference in means do not apply. But we still can find a confidence interval for or perform hypothesis tests about the average difference \(\mu_X - \mu_Y\).

We simply look at the set of differences, \(d_1 = x_1 - y_1, d_2 = x_2 - y_2, \ldots, d_n = x_n - y_n\), and consider it to be a single random sample \(d_1, d_2, \ldots, d_n\) for which we can compute the sample mean difference \(\bar{d}\) and sample difference deviation \(S_d\). Then we compute a confidence interval for the average difference \(\mu_d\) by either

\[
\mu_d = \bar{d} \pm \frac{z_{\alpha/2} S_d}{\sqrt{n}} \quad \text{or} \quad \mu_d = \bar{d} \pm \frac{t_{\alpha/2} S_d}{\sqrt{n}}
\]

(for large samples)

(where \(t_{\alpha/2}\) is the \(t\)-score from the \(t(n-1)\) distribution if the differences appear to be normally distributed).

Hypothesis tests about the average difference \(\mu_d\) can then be made with the standard \(z\)-test or \(t\)-test as appropriate.

Example. A survey was conducted to compare the difference in high school grade point average with the grade point average earned after one year of college. The following data gives the high school GPA and the first year college GPA of the same person from a random sample of students at a university.

(a) Find a 95% confidence interval for the difference in average GPA. Explain the interval in words.

(b) Test the hypothesis that the average high school GPA is no more than 0.6 higher than the average first year college GPA.

\[
\begin{array}{cccccccc}
\text{H.S} & \text{Coll.} & \text{H.S} & \text{Coll.} & \text{H.S} & \text{Coll.} & \text{H.S} & \text{Coll.} \\
2.65 & 1.87 & 3.72 & 1.33 & 3.22 & 1.60 & 3.63 & 3.04 \\
3.14 & 2.27 & 3.25 & 1.13 & 3.01 & 2.37 & 2.48 & 3.00 \\
3.42 & 2.56 & 3.38 & 3.40 & 3.54 & 1.43 & 3.49 & 3.73 \\
3.18 & 2.35 & 2.69 & 1.96 & 3.21 & 2.74 & 3.14 & 3.17 \\
3.40 & 3.02 & 3.54 & 1.96 & 3.17 & 1.95 & 3.03 & 1.57 \\
3.81 & 2.44 & 3.84 & 1.77 & 3.06 & 2.42 & 3.17 & 1.71 \\
\end{array}
\]

Solution. Because the first and second GPAs are from the same person, the measurements are clearly dependent; hence, we will create a new sample by converting the data into one population measuring the high school GPA minus the college GPA.

First, we enter the high school data under \(\text{L1}\) in the \text{STAT Edit} screen, and enter the college data under \(\text{L2}\). Next, enter the command \(\text{L1} - \text{L2} \Rightarrow \text{L3}\) to enter the differences into list \(\text{L3}\). (The \(\Rightarrow\) is the Store command right above the On button.)
(a) To find a confidence interval for the average difference $\mu_d$, we shall assume grades that the grades are normally distributed. The TInterval gives us $0.58 \leq \mu_d \leq 1.1535$. That is, for students at this university, the average high school GPA is from 0.58 points higher to 1.1535 points higher than the average first-year college GPA.

(b) Next we apply a $t$-test to $H_0: \mu_d = 0.6$ with the one-sided alternative $H_a: \mu_d > 0.6$. If $\mu_d = 0.6$ were true, then there would only be a 3.33% chance of obtaining a sample mean difference of $\bar{d} = 0.867$ or higher with a random sample of $n = 30$. We have significant evidence to reject $H_0$ in favor of the alternative.

**Exercise**

The Vitamin C level of various samples of wheat soy blend were measured before and after storage and shipment to Haiti. Here are the results in mg/100 g:

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(a) Test whether or not there is any significant difference in the mean vitamin C level at the factory and later in Haiti.
(b) Find a 95% confidence interval for the mean change in Vitamin C level.
Solution

First enter the factory measurements into list L4 and the measurements from Haiti into list L5. Next use the command L4–L5 ⇒ L6 to enter the differences into list L6.

Since it appears that there is a drop in Vitamin C levels, we shall test $H_0: \mu_d = 0$ with the alternative $H_a: \mu_d > 0$. We now use the T–Test feature on list L6.

We obtain a very low $P$-value of 0.0000187; thus we can say that there is a significant difference in average Vitamin C level. If not, then there would be almost no chance of having a $d$ of 5.33 or higher with this sample of 27 measurements.

(b) Now use the TInterval feature on list L6 to find a 95% confidence interval for $\mu_d$.

Because the confidence interval (3.1226, 7.5441) for the average difference does not contain 0, we have further evidence to reject that $\mu_d = 0$. The average level before shipping is from 3.1226 to 7.5441 (mg/100 g) higher than after shipping.