Bottles of liquid soap are normally distributed in weight with a mean of 64 ounces and a standard deviation 0.5 ounces.

(a) Let various random samples of size 21 be collected. Compute \( P(0.47 \leq S \leq 0.55) \).

With \( n = 21 \) and \( \sigma = 0.5 \):

\[
P(0.47 \leq S \leq 0.55) = P\left( \frac{20 \times 0.47^2}{0.5^2} \leq \frac{(n-1)S^2}{\sigma^2} \leq \frac{20 \times 0.55^2}{0.5^2} \right) = P\left( 17.672 \leq \chi^2(20) \leq 24.2 \right) = 0.37524 .
\]

Use \( \chi^2 \text{cdf}(17.672, 24.2, 20) \)

(b) Suppose \( \sigma \) is unknown, but a random sampling of 26 bottles yields \( S = 0.60 \). Find a 95\% confidence interval for the true standard deviation.

Use the 95\% chi-square scores from the \( \chi^2 (n - 1) = \chi^2 (25) \) curve which are \( L = 13.12 \) and \( R = 40.65 \).

\[
\sqrt{\frac{(n - 1) \times S^2}{R}} \leq \sigma \leq \sqrt{\frac{(n - 1) \times S^2}{L}} \leq \frac{25 \times 0.60^2}{40.65} \leq \sigma \leq \frac{25 \times 0.60^2}{13.12} \leq 0.47 \leq \sigma \leq 0.828
\]

(c) Using \( S = 0.60 \) from a sample of 26 bottles, is there evidence, at the 10\% level of significance, to reject the claim that \( \sigma = 0.5 \)? State null and alternative hypotheses, give the test-statistic and \( P \)-value, and use the \( P \)-value to explain your conclusion in detail.

With \( S = 0.6 \) and \( n = 26 \) : We test \( H_0 : \sigma = 0.50 \) vs. \( H_a : \sigma > 0.50 \).

The test stat is \( \frac{(n - 1)S^2}{\sigma^2} = \frac{25 \times 0.6^2}{0.5^2} = 36 \). The \( P \)-value is \( \chi^2 \text{cdf}(36, 1E99, 25) = 0.0716 \). (Use the right tail for \( H_a : \sigma > 0.50 \).)

If \( \sigma = 0.50 \) were true, then there is a 7.16\% chance of obtaining an \( S \) of 0.60 or larger with a sample of size 26.

There is enough evidence to reject \( H_0 \) with a 10\% level of significance.