1. GE is testing the average lifetimes of light bulbs under normal usage from two production lines. The results from two samples are below:

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>$\bar{x}$ (hrs)</th>
<th>$S$ (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Line 1</td>
<td>400</td>
<td>708.4</td>
<td>16.5</td>
</tr>
<tr>
<td>Production Line 2</td>
<td>500</td>
<td>690.6</td>
<td>16.2</td>
</tr>
</tbody>
</table>

Let $\mu_1$ be the true average lifetime among all light bulbs from Production Line 1 and let $\mu_2$ be the true average lifetime among all light bulbs from Production Line 2.

Use the data to test whether $\mu_1 = \mu_2$.

State $H_0$ and $H_a$.

$H_0$: $\mu_1 = \mu_2$ vs. $H_a$: $\mu_1 > \mu_2$

$\mu_1 - \mu_2 = 0$ vs. $\mu_1 - \mu_2 > 0$

Note: $\bar{x}_1 - \bar{x}_2 = 17.8 > 0$

Show how to compute the test stat.

$z = \frac{17.8}{\sqrt{\frac{16.5^2}{400} + \frac{16.2^2}{500}}} \approx 16.2$

Compare to right-side of $N(0,1)$ curve

Use the $P$-value to explain your conclusion.

The small $P$-value of $2.19 \times 10^{-59}$ shows that we have strong evidence to reject $H_0$.

Use the data to test whether $\mu_1$ is 18 hours more than $\mu_2$.

State $H_0$ and $H_a$.

$H_0$: $\mu_1 - \mu_2 = 18$ vs. $H_a$: $\mu_1 - \mu_2 < 18$

$\mu_1 = \mu_2 + 18$ vs. $\mu_1 < \mu_2 + 18$

Note: $\bar{x}_1 - \bar{x}_2 = 17.8 < 18$

Show how to compute the test stat.

$z = \frac{17.8 - 18}{\sqrt{\frac{16.5^2}{400} + \frac{16.2^2}{500}}} \approx -0.182$

Compare to left-side of $N(0,1)$ curve

Use the $P$-value to explain your conclusion.

The large $P$-value of 0.4277 shows that we have no evidence to reject $H_0$. 
2. Below are sample data on ACT scores for college-bound students from two different states. Let $\mu_1$ be the average ACT score among all college-bound Kentuckians and let $\mu_2$ be the average ACT score among all college-bound Hoosiers.

<table>
<thead>
<tr>
<th></th>
<th>$n$</th>
<th>$\bar{x}$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KY students</td>
<td>450</td>
<td>21.6</td>
<td>2.9</td>
</tr>
<tr>
<td>IN students</td>
<td>550</td>
<td>23.4</td>
<td>2.8</td>
</tr>
</tbody>
</table>

(a) Test the claim that there is no difference in average. State $H_0$, $H_a$, show how to find the test stat, and use the $P$-value to explain your conclusion.

$H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$  
Note that $\bar{x}_1 - \bar{x}_2 = -1.8$

$H_a: \mu_1 < \mu_2$ or $\mu_1 - \mu_2 < 0$  
(because $\bar{x}_1 < \bar{x}_2$ or because $\bar{x}_1 - \bar{x}_2 < 0$)

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \approx -9.917 \rightarrow \text{compare to left-tail of } N(0,1)$$

The $P$-value of 0 shows that we have strong evidence to reject $H_0$.

(b) Test the claim that the KY average is 2 points lower than the IN average. State $H_0$, $H_a$, show how to find the test stat, and use the $P$-value to explain your conclusion.

$H_0: \mu_1 = \mu_2 - 2$ or $\mu_1 - \mu_2 = -2$

$H_a: \mu_1 - \mu_2 > -2$  (because $\bar{x}_1 - \bar{x}_2 = -1.8 > -2$)

$$z = \frac{-1.8 - (-2)}{\sqrt{\frac{2.9^2}{450} + \frac{2.8^2}{550}}} \approx 1.1 \rightarrow \text{compare to right-tail of } N(0,1) \text{ curve}$$

The $P$-value of 0.13525 means that there is not enough evidence to reject $H_0$. 