Summary of Confidence Interval Formulas

<table>
<thead>
<tr>
<th>Level of Conf.</th>
<th>0.90</th>
<th>0.95</th>
<th>0.98</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>z-score = ( \frac{z_{\alpha/2}}{2} )</td>
<td>1.645</td>
<td>1.96</td>
<td>2.326</td>
<td>2.576</td>
</tr>
</tbody>
</table>

For \( c \leq X \leq d \), then \( \sigma \leq U = (d - c) / 2 \). For proportions, \( \sigma = \sqrt{p(1 - p)} \leq 0.5 \).

Sample Size

\[
n \geq \left( \frac{z_{\alpha/2} U}{e} \right)^2, \quad \text{or} \quad n \geq \frac{N \times \left( \frac{z_{\alpha/2} U}{e} \right)^2}{(N - 1) + \left( \frac{z_{\alpha/2} U}{e} \right)^2}
\]

Large Population Confidence Interval for Mean (ZInterval on Calculator)

\[
\mu \approx \bar{x} \pm \frac{z_{\alpha/2} \sigma}{\sqrt{n}}
\]


For a Small Population of Size \( N \)

\[
\mu \approx \bar{x} \pm \frac{z_{\alpha/2} \sigma}{\sqrt{n}} \cdot \frac{\sqrt{N - n}}{\sqrt{N - 1}}
\]

Use \( S \) or \( U \) for \( \sigma \) as appropriate.

Proportion Confidence Interval

\[
p = p \pm \frac{z_{\alpha/2} \sqrt{p(1 - p)}}{\sqrt{n}} \quad (1 - \text{PropZInt on Calc.})
\]

\[
p = p \pm \frac{z_{\alpha/2} \times 0.5}{\sqrt{n}} \quad \text{(most used in practice)}
\]

For small populations of size \( N \), multiply the margin of error by \( \sqrt{\frac{N - n}{N - 1}} \).

Confidence Interval for Mean of Normally Distributed Measurements

\[
\mu \approx \bar{x} \pm \frac{t_{\alpha/2} S}{\sqrt{n}}, \quad (\text{TInterval on Calculator})
\]

where \( t_{\alpha/2} \) is the appropriate \( t \)-score from the \( t(n - 1) \) distribution.

Applies for all sample sizes (not just large samples) and we do not need to know \( \sigma \). But we must be working with normally distributed measurements.
## Two-Sample Confidence Intervals

### Difference of Means

\[ \mu_1 - \mu_2 \approx (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}} \]

*(2-SampZInt on Calc.)*

Arbitrary Measurements: Need large samples.  
Normal Measurements: Any sample size works.

With large samples, we may replace \( \sigma_1 \) and \( \sigma_2 \) with estimates or upper bounds.

Small Populations:  
\[ \mu_1 - \mu_2 \approx (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma^2_1}{n_1} \left( \frac{N_1 - n_1}{N_1 - 1} \right) + \frac{\sigma^2_2}{n_2} \left( \frac{N_2 - n_2}{N_2 - 1} \right)} \]

### Difference of Proportions

\[ p_1 - p_2 \approx (\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2} \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}} \]  
*or use*  
\[ p_1 - p_2 \approx (\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2} \sqrt{\frac{0.25}{n_1} + \frac{0.25}{n_2}} \]  
*(2-PropZInt on Calc.)*

Small Populations:

\[ p_1 - p_2 \approx (\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{n_1} \left( \frac{N_1 - n_1}{N_1 - 1} \right) + \frac{\bar{p}_2(1 - \bar{p}_2)}{n_2} \left( \frac{N_2 - n_2}{N_2 - 1} \right)} \]

*or*  
\[ p_1 - p_2 \approx (\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2} \sqrt{\frac{0.25}{n_1} \left( \frac{N_1 - n_1}{N_1 - 1} \right) + \frac{0.25}{n_2} \left( \frac{N_2 - n_2}{N_2 - 1} \right)} \]

### Difference of Means of Normally Distributed Measurements

\[ \mu_1 - \mu_2 \approx (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{\frac{S^2_1}{n_1} + \frac{S^2_2}{n_2}} \]

*or*  
\[ \mu_1 - \mu_2 \approx (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \]

*(2-SampTInt on Calc.)*

Here,  
\[ S_p = \sqrt{\frac{(n_1 - 1)S^2_1 + (n_2 - 1)S^2_2}{n_1 + n_2 - 2}}. \]