1. Evaluate and simplify \( f'(x) \).

(a) \( f(x) = 4x^3 - 16\sqrt{x} + \frac{8}{x^2} \) 

(b) \( f(x) = \left|x - 8\right| + \frac{4}{\sqrt{x} + 6} - 12(x - 4)^3 + 6x \)

(c) \( f(x) = 12x^{5/3} - 6x^{1/3} - \frac{9}{x^{2/3}} \) 

(d) \( f(x) = \sec(4x) + \tan(4x) \)

2. Evaluate and simplify \( f'(t) \).

(a) \( f(t) = -16t^2 + 64t + 56 \) 

(b) \( f(t) = 6 \cos t + 6 \sin(2t) \) 

(c) \( f(t) = 8t^{3/2} - \frac{4}{t} \)

3. Let \( f(x) = \frac{16}{x} - 3x^2 \).

(a) Find the average rate of change of \( f \) over the interval \([2, 4]\).

(b) Find the exact rate of change of \( f \) at \( x = 2 \).

4. Let \( f(x) = \frac{24}{\sqrt{x}} - 3x^{2/3} \).

(a) Find the average rate of change of \( f \) over the interval \([1, 64]\).

(b) Find the exact rate of change of \( f \) at \( x = 64 \).

5. The average velocity of a moving object is \( \text{Change in Dist}/\text{Change in Time} \). Find your average velocity if you drive 54 miles in 48 minutes.

6. A car travelling at 30 mph begins accelerating. The additional distance travelled after \( t \) hours is given by \( d(t) = 12150t^2 + 30t \) miles.

(a) After 2 seconds, (i) how much further has the car travelled? (ii) What is its speed?

(b) Determine the car’s acceleration in \( \text{ft/sec}^2 \).

7. A Supersonic Rocket Ship blasts out of orbit on the way to Uranus. For the next day, its distance from Earth after \( t \) hours is given by \( d(t) = 3000t^{4/3} + 18000t + 300 \) miles.

(a) What is its initial distance from Earth?

(b) At exactly 8 hours after blasting out of orbit,

(i) How much further from Earth has it travelled? (ii) What is its speed? (iii) What is its acceleration?
8. An object launched at \( v_0 \) ft/s from \( h_0 \) ft at an angle of \( \theta^\circ \) from the horizontal has a vertical height function of \( h(t) = -16t^2 + (v_0 \sin \theta)t + h_0 \) ft. The maximum height occurs when the vertical velocity is 0.

(a) Find the maximum height of an object launched from 6 ft at 40 ft/s at a 30º angle.

(b) Find the maximum height of an object launched straight upward from 6 ft at 40 ft/s.
Answers

1. (a) \( f'(x) = 12x^2 - \frac{8}{\sqrt{x}} - \frac{16}{x^3} \)  
   (b) \( f'(x) = \frac{|x-8|}{x-8} - \frac{2}{(x+6)^{\frac{3}{2}}} - 36(x-4)^2 + 6 \)
   (c) \( f'(x) = 20x^{\frac{2}{3}} - \frac{2}{x^{\frac{2}{3}}} + \frac{6}{x^{\frac{5}{3}}} \)  
   (d) \( f'(x) = 4 \sec(4x) \tan(4x) + 4 \sec^2(4x) \)

2. (a) \( f'(t) = -32t + 64 \)  
   (b) \( f'(t) = -6 \sin t + 12 \cos(2t) \)  
   (c) \( f'(t) = 12\sqrt{t} + \frac{4}{t^2} \)

3. (a) Avg Rate of Change: \( \frac{f(4) - f(2)}{4 - 2} = \frac{-44 - (-4)}{2} = -20 \)
   (b) \( f'(x) = \frac{-16}{x^2} - 6x \); Exact rate of change at \( x = 2 \) is \( f'(2) = -4 - 12 = -16 \)

4. (a) Avg Rate of Change: \( \frac{f(64) - f(1)}{63} = \frac{(3-48) - (24-3)}{63} = \frac{-66}{63} = -\frac{22}{21} \)
   (b) \( f'(x) = \frac{-12}{x^{\frac{3}{2}}} - \frac{2}{x^{\frac{1}{3}}} \); Exact rate of change at \( x = 64 \) is \( f'(64) = -12 / 8^3 - 1/2 = -\frac{67}{128} \)

5. \( \frac{54 \text{ miles}}{48 \text{ hrs}} = 67.5 \text{ mph.} \)

6. (a) Note that 2 seconds is \( 2/3600 \) hours. (i) So the distance travelled in this time is \( d(2/3600) = 12150 (2/3600)^2 + 30 \times 2 / 3600 = 0.020416 \) miles.
   Multiplying by 5280 (ft per mile), we obtain a distance of 107.8 ft.
   (ii) The velocity function is \( v(t) = d'(t) = 24300t + 30 \) mph. After 2 sec., the speed is \( v(2/3600) = 24300 \times 2 / 3600 + 30 = 43.5 \) mph.
   (b) The acceleration function is \( a(t) = v'(t) = d''(t) = 24,300 \) miles per hr\(^2\). Converting to ft / sec\(^2\), we obtain
   \[
a = \frac{24300 \text{ mile}}{\text{hr}^2} \times 5280 \frac{\text{ft}}{\text{mile}} \times \frac{1}{3600^2} \frac{\text{hr}^2}{\text{sec}^2} = 9.9 \frac{\text{ft}}{\text{sec}^2}.
\]
7. (a) The initial distance from Earth is \(d(0) = 300\) miles. (b) After 8 hours, the distance is \(d(8) = 3000 \times 8^{1/3} + 18000 \times 8 + 300 = 192,300\) miles, so the ship has travelled 192,000 more miles from Earth.

The velocity function is \(v(t) = d'(t) = 4000t^{1/3} + 18000\) mph; so after 8 hours, the speed is \(v(8) = 4000 \times 8^{1/3} + 18000 = 26,000\) mph.

The acceleration function is \(a(t) = v'(t) = d''(t) = \frac{4000}{3}t^{-2/3}\) miles per hr\(^2\). After 8 hours, the acceleration is \(a(8) = \frac{4000}{3 \times 8^{2/3}} = 333\frac{1}{3}\) miles per hr\(^2\).

8. (a) \(h(t) = -16t^2 + (40\sin 30^\circ)t + 6 = -16t^2 + 20t + 6\) ft and \(v(t) = -32t + 20\) ft/s.

Then \(v(t) = 0\) at \(t = 20 / 32 = 0.625\) sec. The height at this time is the maximum height and is given by \(h(0.625) = -16(0.625)^2 + 20 \times 0.625 + 6 = 12.25\) ft

(b) For straight upward, we have \(h(t) = -16t^2 + (40\sin 90^\circ)t + 6 = -16t^2 + 40t + 6\) ft and \(v(t) = -32t + 40\) ft/s.

Then \(v(t) = 0\) at \(t = 40 / 32 = 1.25\) sec. The height at this time is the maximum height and is given by \(h(1.25) = -16(1.25)^2 + 40 \times 1.25 + 6 = 31\)