1. (a) \[ \lim_{x \to \frac{7\pi}{6}} 8\cos x = 8 \left( -\frac{\sqrt{3}}{2} \right) = -4\sqrt{3} \]  
(b) \[ \lim_{x \to \frac{5\pi}{3}} 6\sin x = 6 \left( -\frac{\sqrt{3}}{2} \right) = -3\sqrt{3} \]  
(c) \[ \lim_{x \to \frac{2\pi}{3}} 4\cos x = 4 \left( -\frac{1}{2} \right) = -2 \]  
(d) \[ \lim_{x \to \frac{7\pi}{4}} 2\sin x = 2 \left( -\frac{\sqrt{2}}{2} \right) = -\sqrt{2} \]  
(e) \[ \lim_{x \to -\frac{\pi}{2}^+} \tan x = -\infty \quad \lim_{x \to \frac{\pi}{6}} \tan x = -\frac{1}{\sqrt{3}} \quad \lim_{x \to \frac{\pi}{3}} \tan x = \sqrt{3} \]  

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2. (a) \[ \lim_{x \to 5} \frac{-10(x - 5)}{x - 5} = \lim_{x \to 5} (-10) = -10 \]  
(b) \[ \lim_{x \to 6^-} \frac{x + 2}{6 - x} = \frac{8}{-3} = +\infty \quad \text{and} \quad \lim_{x \to 6^+} \frac{x + 2}{6 - x} = \frac{8}{0} = -\infty \]  
(c) \[ \lim_{x \to -3^-} \frac{-4}{x + 3} = \frac{-4}{-3} = +\infty \]  
(d) \[ \lim_{x \to 2^-} \frac{4|\!x - 2|}{x - 2} = -4, \text{ but } \lim_{x \to 2^+} \frac{4|\!x - 2|}{x - 2} = +4; \text{ thus, } \lim_{x \to 2} \frac{4|\!x - 2|}{x - 2} \text{ does not exist.} \]  
(e) \[ \lim_{x \to -6} \frac{20|\!x + 6|}{x + 6} \text{ does not exist because } \lim_{x \to -6^-} \frac{20|\!x + 6|}{x + 6} = -20, \lim_{x \to -6^+} \frac{20|\!x + 6|}{x + 6} = +20 \]  
(f) \[ \lim_{x \to -2^-} \frac{x}{4 - x^2} = \frac{-2}{-3} = +\infty \quad \lim_{x \to -2^+} \frac{x}{4 - x^2} = \frac{-2}{3} = -\infty \]  
(g) \[ \lim_{x \to -\frac{\pi}{2}} \cot x = -\infty \quad \lim_{x \to \frac{\pi}{2}} \cot x = +\infty \]  

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3. (a) \[ \frac{1 + \cot(-\frac{\pi}{4})}{0} = \frac{1 + -1}{0} = \frac{0}{0} \]  
(b) \[ \infty \times \tan 0 = \infty \times 0 \]  
(c) \[ \frac{\ln 1}{0} = \frac{0}{0} \]  
(d) \[ \infty^0 = \infty^0 \]  
(e) \[ 0^\cot(\frac{\pi}{2}) = 0^0 \]  
(f) \[ 1^\infty \]
4. All limits are of the form 0/0. So first algebraically manipulate the function and then re-evaluate the limit:

(a) \[
\frac{3 - \sqrt{x + 13}}{x^3 + 64} = \frac{3 - \sqrt{x + 13}}{(x + 4)(x^2 - 4x + 16)} \times \frac{3 + \sqrt{x + 13}}{3 + \sqrt{x + 13}}
\]
\[
= \frac{3 - \sqrt{x + 13}}{(x + 4)(x^2 - 4x + 16)} \times \frac{9 - (x + 13)}{3 + \sqrt{x + 13}}
\]
\[
= \frac{-x - 4}{(x + 4)(x^2 - 4x + 16)} \times \frac{3 + \sqrt{x + 13}}{3 + \sqrt{x + 13}}, \text{ for } x \neq -4
\]
Re-do: \[
\lim_{x \to -4} \frac{-1}{(x^2 - 4x + 16)(3 + \sqrt{x + 13})} = \frac{-1}{(16 + 16 + 16)(3 + \sqrt{9})} = \frac{-1}{48 \times 6}.
\]

(b) Since \(-16\) is a root of the denominator, \(x - (-16) = x + 16\) is a factor.

\[
\frac{1}{\sqrt{-x} - 4} = \frac{4}{(x + 16)(6x - 5)} - \frac{\sqrt{-x}}{(x + 16)(6x - 5)} = \frac{4}{\sqrt{-x}} - \frac{\sqrt{-x}}{(x + 16)(6x - 5)}
\]
\[
= \frac{4 - \sqrt{-x}}{\sqrt{-x} (x + 16)(6x - 5)} \times \frac{4 + \sqrt{-x}}{4 + \sqrt{-x}} = \frac{16 - (-x)}{4 \sqrt{-x} (x + 16)(6x - 5)(4 + \sqrt{-x})}
\]
\[
= \frac{1}{4 \sqrt{-x} (6x - 5)(4 + \sqrt{-x})}, \text{ for } x \neq -16
\]
Re-do:
\[
\lim_{x \to -16} \frac{1}{4 \sqrt{-x}(6x - 5)(4 + \sqrt{-x})} = \frac{1}{4 \times 4 \times (-101) \times 8} = \frac{1}{16 \times (-808)}
\]

(c) \[
\frac{x^2 - 81}{3 - \sqrt{x}} = \frac{(x - 9)(x + 9)}{3 - \sqrt{x}} \times \frac{3 + \sqrt{x}}{3 + \sqrt{x}} = \frac{(x - 9)(x + 9)(3 + \sqrt{x})}{9 - x} = -(x + 9)(3 + \sqrt{x}),
\]
for \(x \neq 9\). Then \[
\lim_{x \to 9} -(x + 9)(3 + \sqrt{x}) = -(18)(6) = -108.
\]
5. (a) (i) \( \lim_{x \to -3^-} f(x) = 3 \) and \( \lim_{x \to -3^+} f(x) = 3 \), the same from either side, so \( \lim_{x \to -3} f(x) = 3 \) also.

(ii) But \( f(-3) = 4 \). So \( f \) is not continuous at \( x = -3 \) since \( \lim_{x \to -3} f(x) \neq f(-3) \).

(iii) There is a “hole” in the graph (removable discontinuity), and \( f \) is neither left-continuous nor right-continuous at \( x = -3 \) since \( \lim_{x \to -3^-} f(x) \neq f(-3) \neq \lim_{x \to -3^+} f(x) \).

(b) (i) \( \lim_{x \to \pi/2^-} g(x) = +\infty \) and \( \lim_{x \to \pi/2^+} g(x) = 1 \), different from either side so \( \lim_{x \to \pi/2} g(x) \) does not exist.

(ii) \( g \) is not continuous at \( x = \pi / 2 \) since \( \lim_{x \to \pi/2} g(x) \) does not exist.

(iii) There is a vertical asymptote on from the left side (infinite discontinuity). However since \( \lim_{x \to \pi/2^+} g(x) = g(\pi / 2) \), then \( g \) is right-continuous at \( x = \pi / 2 \).

(c) (i) \( \lim_{x \to -8^-} h(x) = 4 \) and \( \lim_{x \to -8^+} h(x) = 2 \), different from either side so \( \lim_{x \to -8} h(x) \) does not exist.

(ii) \( h \) is not continuous at \( x = -8 \) since \( \lim_{x \to -8} h(x) \) does not exist.

(iii) There is a (non-removable) jump discontinuity at \( x = -8 \). However \( h \) is left-continuous at \( x = -8 \) since \( \lim_{x \to -8^-} h(x) = 4 = h(-8) \).

(d) Since \( \lim_{x \to 1^-} k(x) = -2 \) and \( \lim_{x \to 1^+} k(x) = -2 \) are the same from either side, then \( \lim_{x \to 1} k(x) = -2 \) also. And \( k(1) = -2 \), so \( k \) is continuous at \( x = 1 \) since \( \lim_{x \to 1} k(x) = k(1) \).

Note: \( k \) is both left-continuous and right-continuous at \( x = 1 \) since

\[
\lim_{x \to 1^-} k(x) = k(1) = \lim_{x \to 1^+} k(x).
\]
6. (a) \( B(x) = 18.10 + 0.66x \), for \( x \geq 0 \).

(i) \( \lim_{{x \to 0^+}} B(x) = 18.10 \)  As the ccf of gas used decreases to 0, then the gas bill decreases to $18.10.

(ii) \( \lim_{{x \to 100^-}} B(x) = 84.10 \)  As the ccf of gas used increases to 100, then the gas bill increases to $84.10.

(b) \( \lim_{{p \to 0^+}} \phi(p) = 1 \); as the probability of success on any try decreases to 0, then the average number of attempts until the first failure decreases to 1.

\( \lim_{{p \to 1^-}} \phi(p) = +\infty \); as the probability of success on any try increases to 1, then the average number of attempts until the first failure increases to \( \infty \).

(c) \( \lim_{{p \to 0^+}} \Omega(p) = 0 \), as the probability of success on any try decreases to 0, then the chance of success within 10 tries also decreases to 0.

\( \lim_{{p \to 1^-}} \Omega(p) = 1 \), as the probability of success on any try increases to 1, then the chance of success within 10 tries also increases to 1.