1. Suppose \( x^2 - 8 \leq f(x) \leq 4 \cos\left(\frac{\pi x}{2}\right) \), for \(-1 \leq x \leq 2\). Graph the outer two functions on the interval. Then explain which one-sided limit can be determined and compute this limit.

2. Suppose \( x^2 - \frac{11}{2} \leq f(x) \leq 3 \sin\left(\frac{\pi x}{12}\right) \), for \(-2 \leq x \leq 2\). Graph the outer two functions on the interval. Then explain which one-sided limit can be determined and compute this limit.

3. Suppose there is some function \( f(x) \) such that \( \sqrt{x + 5} \leq f(x) \leq (x - 4)^2 + 3 \) for all \( x \). What can you conclude from the graphs?
Solutions

1. Because \( \lim_{x \to 2^-} (x^2 - 8) = -4 \) and \( \lim_{x \to 2^-} 4 \cos(\pi x / 2) = -4 \), and
   \[ x^2 - 8 \leq f(x) \leq 4 \cos\left(\frac{\pi x}{2}\right) \]
   on the interval, by the Squeezing Theorem \( \lim_{x \to 2^-} f(x) = -4 \) also.

   Note that \( \lim_{x \to -1^+} f(x) \) cannot be determined since the two outer functions have different one-sided limits as \( x \) decreases to \(-1\).

2. Because \( \lim_{x \to -2^+} \left(\frac{x^2}{2} - \frac{11}{2}\right) = -1.5 \) and \( \lim_{x \to -2^+} 3 \sin\left(\frac{\pi x}{12}\right) = -1.5 \), and
   \[ \frac{x^2}{2} - \frac{11}{2} \leq f(x) \leq 3 \sin\left(\frac{\pi x}{12}\right) \]
   on the interval, by the Squeezing Theorem \( \lim_{x \to -2^+} f(x) = -1.5 \) also.

   Note that \( \lim_{x \to 2^-} f(x) \) cannot be determined since the two outer functions have different one-sided limits as \( x \) increases to \(2\).

3. Because \( \lim_{x \to 4} \sqrt{x + 5} = 3 = \lim_{x \to 4} (x - 4)^2 + 3 \), and
   \[ \sqrt{x + 5} \leq f(x) \leq (x - 4)^2 + 3 \]
   for all \( x \), by the Squeezing Theorem, we can conclude that \( \lim_{x \to 4} f(x) \) exists and equals 3.