Evaluate the specified limits and completely describe the behavior in words in terms of how the variable and function values increase/decrease:

1. (a) \( \lim_{x \to \pi/4^-} 3 \tan x \) 
   (b) \( \lim_{x \to \pi/4^+} 3 \tan x \) 
   (c) \( \lim_{x \to \pi/4} 3 \tan x \)

2. (a) \( \lim_{x \to \pi/3^-} 8 \cos x \) 
   (b) \( \lim_{x \to \pi/3^+} 8 \cos x \) 
   (c) \( \lim_{x \to \pi/3} 8 \cos x \)

3. (a) \( \lim_{x \to 0^-} (9 - x^2) \) 
   (b) \( \lim_{x \to 0^+} (9 - x^2) \) 
   (c) \( \lim_{x \to 0} (9 - x^2) \)

4. (a) \( \lim_{x \to -4^-} -\sqrt{25 - x^2} \) 
   (b) \( \lim_{x \to -4^+} -\sqrt{25 - x^2} \) 
   (c) \( \lim_{x \to -4} -\sqrt{25 - x^2} \)

Evaluate the specified limits:

5. Let \( f(x) = \begin{cases} 
-4 \cos(\pi x) & \text{if } x < 4/3 \\
-2 & \text{if } x = 4/3 \\
4 \sin(\pi x/8) & \text{if } x > 4/3. 
\end{cases} \)

   (a) \( \lim_{x \to 4/3^-} f(x) \) 
   (b) \( \lim_{x \to 4/3^+} f(x) \) 
   (c) \( \lim_{x \to 4/3} f(x) \)

6. (a) \( \lim_{x \to -10^-} \frac{8|x + 10|}{x + 10} \) 
   (b) \( \lim_{x \to -10^+} \frac{8|x + 10|}{x + 10} \) 
   (c) \( \lim_{x \to -10} \frac{8|x + 10|}{x + 10} \)

Describe the behavior of the one-sided limits:

7. (a) \( \lim_{x \to 5^-} \frac{2x}{5-x} \) 
   (b) \( \lim_{x \to 5^+} \frac{2x}{5-x} \)

8. (a) \( \lim_{x \to -3^-} \frac{3x}{x^2 - 9} \) 
   (b) \( \lim_{x \to -3^+} \frac{3x}{x^2 - 9} \)

9. (a) \( \lim_{x \to \pi^-} \cot x \) 
   (b) \( \lim_{x \to \pi^+} \cot x \)
Answers

1. (a) \( \lim_{x \to \pi/4^-} 3 \tan x = 3 \tan(\pi/4) = 3 \)
   As \( x \) increases to \( \pi/4 \), then \( 3 \tan x \) increases to 3.

   (b) \( \lim_{x \to \pi/4^+} 3 \tan x = 3 \)
   As \( x \) decreases to \( \pi/4 \), then \( 3 \tan x \) decreases to 3.

   (c) \( \lim_{x \to \pi/4} 3 \tan x = 3 \)
   As \( x \) approaches \( \pi/4 \), then \( 3 \tan x \) approaches 3.

2. (a) \( \lim_{x \to \pi/3^-} 8 \cos x = 8 \cos(\pi/3) = 4 \)
   As \( x \) increases to \( \pi/3 \), then \( 8 \cos x \) decreases to 4.

   (b) \( \lim_{x \to \pi/3^+} 8 \cos x = 4 \)
   As \( x \) decreases to \( \pi/3 \), then \( 8 \cos x \) increases to 4.

   (c) \( \lim_{x \to \pi/3} 8 \cos x = 4 \)
   As \( x \) approaches \( \pi/3 \), then \( 8 \cos x \) approaches 4.

3. (a) \( \lim_{x \to 0^-} (9 - x^2) = 9 \)
   As \( x \) increases to 0, then \( 9 - x^2 \) increases to 9.

   (b) \( \lim_{x \to 0^+} (9 - x^2) = 9 \)
   As \( x \) decreases to 0, then \( 9 - x^2 \) increases to 9.

   (c) \( \lim_{x \to 0} (9 - x^2) = 9 \)
   As \( x \) approaches 0, then \( 9 - x^2 \) approaches 9.

4. (a) \( \lim_{x \to -4^-} -\sqrt{25 - x^2} = -3 \)
   As \( x \) increases to \(-4\), then \(-\sqrt{25 - x^2} \) decreases to \(-3\).

   (b) \( \lim_{x \to -4^+} -\sqrt{25 - x^2} = -3 \)
   As \( x \) decreases to \(-4\), then \(-\sqrt{25 - x^2} \) increases to \(-3\).

   (c) \( \lim_{x \to -4} -\sqrt{25 - x^2} = -3 \)
   As \( x \) approaches \(-4\), then \(-\sqrt{25 - x^2} \) approaches \(-3\).
5. (a) \[ \lim_{x \to 4/3^-} f(x) = \lim_{x \to 4/3^-} -4 \cos(\pi x) = -4 \cos(4\pi / 3) = 2 \]

(b) \[ \lim_{x \to 4/3^+} f(x) = \lim_{x \to 4/3^+} 4 \sin(\pi x / 8) = 4 \sin(\pi / 6) = 2 \]

(c) Because \[ \lim_{x \to 4/3^-} f(x) = \lim_{x \to 4/3^+} f(x) = 2, \] then \[ \lim_{x \to 4/3} f(x) = 2 \] also.

Note: The fact that \[ f(4/3) = -2 \] does not matter for the limit in (c).

Because \[ \lim_{x \to 4/3^-} f(x) \neq f(4/3), \] there is a “hole in this graph” at \( x = 4/3 \).

6. (a) \[ \lim_{x \to -10^-} \frac{8|x + 10|}{x + 10} = -8 \]

(b) \[ \lim_{x \to -10^+} \frac{8|x + 10|}{x + 10} = +8 \]

(c) \[ \lim_{x \to -10} \frac{8|x + 10|}{x + 10} \] does not exist since due to the differing one-sided limits. There is a jump discontinuity at \( x = -10 \).

7. (a) \[ \lim_{x \to 5^-} \frac{2x}{5 - x} = \frac{10}{0} = +\infty \]

As \( x \) increases to 5, then \( \frac{2x}{5 - x} \) increases to +\( \infty \).

(b) \[ \lim_{x \to 5^+} \frac{2x}{5 - x} = \frac{10}{-0} = -\infty \]

As \( x \) decreases to 5, then \( \frac{2x}{5 - x} \) decreases to -\( \infty \).

Note that \( \lim_{x \to 5} \frac{2x}{5 - x} \) does not exist due to the asymptote at \( x = 5 \) and due to the differing one-sided limits.

8. (a) \[ \lim_{x \to -3^-} \frac{3x}{x^2 - 9} = \frac{-9}{0} = -\infty \]

As \( x \) increases to -3, then \( \frac{3x}{x^2 - 9} \) decreases to -\( \infty \).

(b) \[ \lim_{x \to -3^+} \frac{3x}{x^2 - 9} = \frac{-9}{-0} = +\infty \]

As \( x \) decreases to -3, then \( \frac{3x}{x^2 - 9} \) increases to +\( \infty \).

\[ \lim_{x \to -3} \frac{3x}{x^2 - 9} \] does not exist due to the asymptote and due to the differing one-sided limits.

9. (a) \[ \lim_{x \to \pi^-} \cot x = -\infty \]

As \( x \) increases to \( \pi \), \( \cot x \) decreases to -\( \infty \).

(b) \[ \lim_{x \to \pi^+} \cot x = +\infty \]

As \( x \) decreases to \( \pi \), \( \cot x \) increases to +\( \infty \).