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AN ANALYTICAL SOLUTION TO ONE-DIMENSIONAL THERMAL CONDUCTION-CONVECTION IN SOIL

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The equation of one-dimensional thermal conduction and convection in soil is solved analytically by applying the traditional harmonic method (HM) and the Laplace transform method (LTM). A simple method to determine accurate values of soil heat diffusivity and liquid water flux density is given. Using this method, we determined the soil thermal diffusivity, $k = 0.85 \times 10^{-6}$ m^2 s^{-1}, and liquid water flux density, $W = 4.3 \times 10^{-6}$ m^3 s^{-1} m^{-2}, for the Naqu site in the summer of 1998. An experimental evaluation of the proposed methods is also given. (Soil Science 2003;168:99–107)

Key words: Analytical solution, soil, thermal conduction, thermal convection.

QUANTIFIED understanding of land-air interactions is of great importance in the study of global energy and water cycles. Much attention has been focused on the examination of energy partitioning and budget (Kim et al., 2000a and b) and the hydrological cycle (Nagai et al., 2000), but few presentations have been made regarding required soil thermal properties. While hydrometeorological modelers focus on distinguishing conductive processes from convective processes in soil, micrometeorologists usually consider only apparent soil thermal diffusivity in their models. Micrometeorologists describe atmospheric motion in much detail, but their descriptions of hydrographic processes lack sophistication. As a result the modeling of energy components and surface temperature by some land surface schemes are disappointing. For example, for reasons that are not yet clear, the Simple Biosphere Model version 2 (SiB2) tends to overestimate sensible heat flux and surface temperature while underestimating latent heat flux (Zhang, et al., 1996; Schelde et al., 1997; Doran et al., 1998). It is, therefore, crucial to determine a better way to describe soil thermal processes.

The distribution of soil temperature and moisture content are key variables in the investigation of soil thermal properties. Areas of concern include thermal conductivity, thermal diffusivity, and volumetric heat capacity. Volumetric heat capacity can be derived easily from soil components (Van Wijk, 1963). Thermal conductivity and thermal diffusivity are related by volumetric heat capacity, and, thus, only one of them needs to be determined. Generally speaking, the soil thermal diffusivity (or apparent diffusivity) should be estimated because it describes the transient process of heat conduction based on temperature boundary conditions. Soil heat transfer is caused by a complex combination of conductive processes and intraporous convective processes (Passerat de Silans et al., 1996). Passerat de Silans et al. (1996) preferred to consider soil thermal diffusion as a bulk process which was assimilated to a conductive process. Several methods of determining apparent soil thermal diffusivity and conductivity have been published. Some involved theoretical models (de Vries, 1963) or semiempirical models (Johansen, 1975). Most resulted from the analytical solution of the one-dimensional heat conduction equation with constant diffusivity in a semi-infinite medium (Horton et al., 1983) because they were applied to homogeneous soil. Horton et al. (1983) examined several of the methods under the assumption that the temperature at the

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upper boundary is well described by a sinusoidal function or by a Fourier series. They showed that the Harmonic method (HM) is more reliable than the other methods examined. The analytical solution used in these methods does not require knowledge of the initial temperature profile because the hypothesis of a constant initial temperature profile is not always fulfilled, particularly in regions where abrupt climatic changes may occur in a short period of time as a result of events such as encountering a cold front. Other authors have used methods based on the Laplace Transform (LTM) that require a constant initial temperature profile (Kavianipoor and Beck, 1977; Asrar and Kanemasu, 1982). Passerat de Silans et al. (1996) summarized these studies and examined them with data from HAPEX-Sahel experiment. It is likely that both HM and LTM make it possibile to obtain apparent soil thermal diffusivity from a measured soil temperature profile with the aid of the least-squares method.

After developing an analytical solution for the heat conduction-convection equation by Fourier transformation, Shao et al. (1998) compared the results from the analytical solution with the data from a field infiltration experiment with natural temperature variations and found good agreement. Ren et al. (2000) presented a method to determine soil water flux and pore water velocity by a heat-pulse technique. This method improved earlier methods by reducing distortion of the water flow field and minimizing heat-induced soil water redistribution. These studies represent the latest developments in determining soil temperature distribution and water flux.

The objectives of this study are (i) to solve the soil thermal conduction-convection equation by applying the traditional harmonic method (HM) and the Laplace transform method (LTM); (ii) to present a simple method to determine accurately values of soil heat diffusivity and liquid water flux density; and (iii) to evaluate the proposed methods experimentally.

DERIVATION OF EQUATIONS

Heat can be transferred in the soil by conduction, convection, and radiation (Rybach and Muffler, 1981); however, most soil temperature changes occur within a shallow layer near the surface (Stull, 1988) where the radiative components may be neglected.

Thermal Conduction

Inasmuch as molecular conduction dominates the transport process, the subsurface heat flux Q_g (W m^{-2}) at depth z in the soil can be described by Fourier's law of heat conduction in a homogeneous body. That is, the flux depends on the soil temperature gradient as follows:

$$Q_g = -\lambda \partial T/\partial z, \tag{1}$$

where the negative sign denotes that heat flows down the temperature gradient $\partial T/\partial z$, λ is the soil thermal molecular conductivity (W m^{-1} K^{-1}) of the soil, T is the soil temperature (K), and z is the vertical coordinate (positive down) in the soil. For a thin layer of the soil of thickness Δz, neglecting any horizontal conduction of heat in the soil (Garratt, 1992), the change in soil temperature with time is governed by the difference in the heat fluxes flowing in and out of the layer. The relationship between Q_0, the surface soil heat flux, and Q_g, the heat flux at a small depth in the soil, can be found by the study of conservation of heat in this shallow layer. The second law of thermodynamics yields the simple prognostic equation:

$$C_g \partial T/\partial t = -\partial Q_g/\partial z, \tag{2}$$

where the left side term denotes the time-dependent rate of change of energy in a unit volume of soil; the right side term is the net input of energy per unit volume as a result of the molecular conduction in the soil; C_g is the soil volumetric heat capacity (J m^{-3} K^{-1}), $C_g = \rho c$, ρ is the soil density (kg m^{-3}) and c is soil specific heat (J kg^{-1} K^{-1}); and t is the time in seconds.

We assumed that soil in the research domain has the properties of (i) isotropy and homogeneity, and (ii) depth-independent water content or its variation showing negligible effect on C_g and λ. In addition, it was assumed that energy exchange occurred only in a vertical direction and that Eq. (2) could be simplified to the equation for thermal conduction inside a solid column, viz.,

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2}, \tag{3}$$

where $k = \lambda / C_g$ (m^2 s^{-1}) is the soil heat diffusivity.

Thermal Convection

Letting Q_v represent the heat flux caused by the vertical movement of water, and assuming the liquid infiltration rate (m^3 s^1 m^2) is w (positive up), Q_v can be expressed as:

$$Q_v = C_w w \varphi \Delta T, \tag{4}$$

where C_w is the heat capacity of water, φ is the water content of the soil, and ΔT is the vertical difference of water temperature in the soil (Fan and Tang, 1994). According to the second law of thermodynamics,

$$C_g \partial T/\partial t = -\partial Q_v/\partial z, \tag{5}$$

where the left side term denotes the time-dependent change rate of energy in a unit volume of soil, and the right side term is the net input of energy per unit volume as a result of the vertical movement of water. Combining Eqs. (4) and (5), we obtain:

$$\partial T/\partial t = \frac{C_w}{C_g} w\varphi \partial T/\partial z, \tag{6}$$

letting $W = C_w/C_g w\varphi$, Eq. (6) can be rewritten as

$$\partial T/\partial t = W\partial T/\partial z, \tag{7}$$

where W is usually considered to be the liquid water flux density ($m^3\ s^{-1}\ m^{-2}$).

Incorporation of Thermal Conduction and Convection

Because the processes of thermal conduction and convection are independent, it is reasonable to incorporate thermal conduction Eq. (3) and convection Eq. (7) to consider these two processes together. This gives:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} + W\frac{\partial T}{\partial z}. \tag{8}$$

SOLUTIONS OF EQUATIONS

Application of the Harmonic Method

Hillel (1982) took diurnal forcing as a purely sinusoidal forcing function, $T(t)|_{z=0} = T_0 + A\sin \omega t$, $t > 0$, where T_0 is a constant, A is the amplitude of the variation of the soil surface temperature, and ω is the angular velocity of the Earth's rotation. $\omega = 2\pi/P = 7.292 \times 10^{-5}$ rads^{-1}, where P ($P = 24 \times 3600$ seconds) is the harmonic period of the surface temperature. Under this boundary condition, he solved Eq. (3); however, he did not account for the convection process.

Given the same boundary condition, the soil thermal conductive equation with the thermal convection term included may be written as:

$$\begin{cases} \dfrac{\partial T}{\partial t} = k\dfrac{\partial^2 T}{\partial z^2} + W\dfrac{\partial T}{\partial z} & (t > 0 \quad z > 0). \\ T|_{z=0} = T_0 + A\sin \omega t & (t \geq 0) \end{cases} \tag{9}$$

Expanding the harmonic method, and setting $T(z, t) = T_0 + u(z, t)$, where $u(z, t)$ is a solution to Eq. (10),

$$\begin{cases} \dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial z^2} + W\dfrac{\partial u}{\partial z} & (t > 0 \quad z > 0). \\ u|_{z=0} = A\sin \omega t & (t \geq 0) \end{cases} \tag{10}$$

It is apparent that there should be a complex exponential solution to Eq. (10), assuming

$$u(z,t) = Ae^{(az+bt)}, \tag{11}$$

where a and b are complex constants. Because $u|_{z=0} = A \sin \omega t$, and $Ue^{bt} = A \sin \omega t$, it is apparent that $A \sin \omega t$ is the imaginary part of Ae^{bt}, resulting in $b = i\omega$.

In order to determine a and b, we put $u(z, t) = Ae^{(az+bt)}$ into $(\partial u/\partial t) = k(\partial^2 u/\partial z^2) + W(\partial u/\partial z)$ of Eq. (10), and then find that $Abe^{(az+bt)} = kAa^2e^{(az+bt)} + WAae^{(az+bt)}$, resulting in $b = ka^2 + Wa$. Combining $b = ka^2 + Wa$ to $b = i\omega$, we obtain this equation in terms of $a : ka^2 + Wa - i\omega = 0$. The solutions are $a = (-W \pm W^2 + 4ik\omega)/(2k)$, considering that a should be negative because the amplitude of the soil temperature decreases with increasing depth. We think $a = (-W - W^2 + 4ik\omega)/(2k)$ is right and $a = (-W^2 + 4ik\omega)/(2k)$ is wrong physically.

Setting $W^2 + 4ik\omega = \alpha + \beta i$ where α and β are real, we obtained

$$\begin{aligned} \alpha &= \sqrt{\frac{W^2+\sqrt{W^4+16k^2\omega^2}}{2}} \\ \beta &= \frac{2\sqrt{2}k\omega}{\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}}} \end{aligned} \tag{12}$$

so $$a = (-W-\sqrt{(W^2+\sqrt{W^4+16k^2\omega^2})/2})/(2k) - \frac{\sqrt{2}\omega}{\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}}}i. \tag{13}$$

Putting Eq. (13) and $b = i\omega$ into Eq. (11), we find that

$$u(z,t) = Ae^{\left[(\frac{W}{2k}\frac{\sqrt{2}}{4k}\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}})z\right]} e^{\left[\omega t - z\frac{\sqrt{2}\omega}{\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}}}\right]i}, \tag{14}$$

Considering the boundary condition as a sinusoidal function, the imaginary part of Eq. (14) should be the solution to Eq. (10). We find

$$u(z,t)=Ae^{\left[(\frac{W}{2k}\frac{\sqrt{2}}{4k}\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}})z\right]}$$
$$\sin(\omega t-\sqrt{2}\omega z/\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}}). \quad (15)$$

Thus the solution to Eq. (9) is

$$T(z,t)=T_0+A\exp\left[(-\frac{W}{2k}-\frac{\sqrt{2}}{4k}\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}})z\right]$$
$$\cdot\sin\left[\omega t-z\frac{\sqrt{2}\omega}{\sqrt{W^2+\sqrt{W^4+16k^2\omega^2}}}\right]. \quad (16)$$

Equation (16) holds for a semi-infinite medium with an upper boundary condition given by a sine function of time. The soil thermal diffusivity k and the convective velocity W can be obtained using measured soil temperature at two different layers at depths z_1 and z_2. The formula is:

$$\begin{cases} k=\dfrac{(z_1-z_2)^2\omega\ln(A_2/A_1)}{(\Phi_2-\Phi_1)[(\Phi_2-\Phi_1)^2+(\ln(A_2/A_1))^2]} \\ W=\dfrac{\omega(z_2-z_1)}{\Phi_2-\Phi_1}\left[1-\dfrac{2(\ln(A_2/A_1))}{(\Phi_2-\Phi_1)^2+(\ln(A_2/A_1))^2}\right] \end{cases} \quad (17)$$

where A_1, A_2 are amplitudes, and Φ_1, Φ_2 are initial phases of soil temperatures at the depths of z_1 and z_2 in soil (Fan and Tang, 1994).

Application of the Laplace Transformation Method

It has become traditional in micrometeorological and hydrographic research to assume that the initial soil profile is constant rather than to varying linearly with depth. Garratt (1992) gave an idealized variation of soil temperature through a diurnal cycle for several depths in the soil. He assumed that mean soil temperature at all levels in soil is uniform by regarding it as the temperature of deep soils; i.e., $\gamma = 0.0$. This assumption has been used widely (e.g., Passerat de Silans et al., 1996). However, in reality, mean soil temperature may vary somewhat with increasing soil depth. We set the initial condition $T|_{t=0} = T_0 - \gamma z$, $(z \geq 0)$, where T_0 is the surface mean temperature (K), γ is the soil temperature lapse rate (K m^{-1}), $\gamma = (\sum_{i=1}^{n} T_{0i} - \sum_{i=1}^{n} T_i/z$ and $\gamma = 0.0$ is the idealized situation, and z is the soil depth (m). Setting the boundary condition the same as above, we obtain:

$$\begin{cases} \dfrac{\partial T}{\partial t}=k\dfrac{\partial^2 T}{\partial z^2}+W\dfrac{\partial T}{\partial z} & (t>0 \quad z>0) \\ T|_{t=0}=T_0-\gamma z & (z\geq 0) \\ T|_{z=0}=T_0+A\sin\omega t & (t\geq 0) \end{cases} \quad (18)$$

We solve Eq. (18) in terms of the Laplace transform scheme by setting $T = T^* + T_0 - \gamma z - \gamma Wt$, and we get

$$\frac{\partial T}{\partial t}=\frac{\partial T^*}{\partial t}-\gamma W,$$
$$\frac{\partial T}{\partial z}=\frac{\partial T^*}{\partial z}-\gamma,$$
$$\frac{\partial^2 T}{\partial z^2}=\frac{\partial^2 T^*}{\partial z^2}.$$

Thus, $$\frac{\partial T^*}{\partial t}-\gamma W=k\frac{\partial^2 T^*}{\partial z^2}+W(\frac{\partial T^*}{\partial z}-\gamma).$$

That is, $$\begin{cases} \dfrac{\partial T^*}{\partial t}=k\dfrac{\partial^2 T^*}{\partial z^2}+W\dfrac{\partial T^*}{\partial z} \\ \textit{for} \quad t=0 \quad T^*=0 \\ \textit{for} \quad z=0 \quad T^*=A\sin\omega t+\gamma Wt \end{cases}. \quad (19)$$

for $$\frac{\partial T^*}{\partial t}=k\frac{\partial^2 T^*}{\partial z^2}+W\frac{\partial T^*}{\partial z}.$$

Setting $$T^{**}=T^*\cdot e^{\frac{W}{2k}z+\frac{W^2}{4k}t},$$

gives

$$\frac{\partial T^{**}}{\partial t}=(\frac{\partial T^*}{\partial t}+\frac{W^2}{4k}T^*)e^{\frac{W}{2k}z+\frac{W^2}{4k}t},$$
$$\frac{\partial T^{**}}{\partial z}=(\frac{\partial T^*}{\partial z}+\frac{W}{2k}T^*)e^{\frac{W}{2k}z+\frac{W^2}{4k}t}.$$
$$\frac{\partial^2 T^{**}}{\partial z^2}=[(\frac{\partial^2 T^*}{\partial z^2}+\frac{W}{2k}\frac{\partial T^*}{\partial z})+\frac{W}{2k}(\frac{\partial T^*}{\partial z}+\frac{W}{2k}T^*)]e^{\frac{W}{2k}z+\frac{W^2}{4k}t}$$
$$=(\frac{\partial^2 T^*}{\partial z^2}+\frac{W}{k}\frac{\partial T^*}{\partial z}+\frac{W^2}{4k^2}T^*)e^{\frac{W}{2k}z+\frac{W^2}{4k}t}.$$

It follows that $$\frac{\partial T^{**}}{\partial t}=k\frac{\partial^2 T^{**}}{\partial z^2}.$$

Consequently, we get a definite problem in the form

$$\begin{cases} \dfrac{\partial T^{**}}{\partial t}=k\dfrac{\partial^2 T^{**}}{\partial z^2} \\ t=0 \quad T^{**}=0 \\ z=0 \quad T^{**}=(A\sin\omega t+\gamma Wt)e^{\frac{W^2}{4k}t} \end{cases}. \quad (20)$$

We now make the Laplace transformation of T^{**} and set $\bar{T} = \mathcal{L}[T^{**}]$ so that

$$p\overline{T} = k\,\frac{d^2\overline{T}}{dz^2},$$

at $z = 0$, $\overline{T} = \mathcal{L}[A \sin \omega t + \gamma Wt)e(w^2/4k)]$, and $T^*|_{z\to\infty}$ is a bounded function. We have

$$\frac{d^2\overline{T}}{dz^2} - \frac{p}{k}\,\overline{T} = 0,$$

and

$$\overline{T} = c_1 e^{\sqrt{\frac{p}{k}}z} + c_2 e^{-\sqrt{\frac{p}{k}}z},$$

From $z \to +\infty$, meaning $\overline{T}$ is bounded, we get $c_1 = 0$.

From $z = 0$, we obtain $c_2 = \mathcal{L}[(A \sin \omega t + \gamma \omega t)e^{(w^2/4k)t)}]$.

As a result, $\overline{T} = \mathcal{L}[(A \sin \omega t + \gamma \omega t)\cdot e^{(w^2/4k)}]\cdot e^{(-z\, p/k)}$.

From retrieval it follows that

$$T^{**} = (A \sin\omega t + \gamma Wt)e^{\frac{W^2}{4k}t} * \mathcal{L}^{-1}[e^{-z\sqrt{\frac{p}{k}}}]$$

$$= (A \sin\omega t + \gamma Wt)e^{\frac{W^2}{4k}t} * \frac{z}{2\sqrt{k\pi}t^{3/2}}\, e^{\frac{z^2}{4kt}},$$

$$= \frac{z}{2\sqrt{k\pi}} \int_0 \frac{(A\sin\omega\tau + \gamma W\tau)\cdot e^{\frac{W^2}{4k}\tau}}{(t-\tau)^{3/2}} \cdot e^{\frac{z^2}{4k(t-\tau)}}\, d\tau$$

$$T^* = \frac{z e^{\frac{W}{2k}z - \frac{W^2}{4k}t}}{2\sqrt{k\pi}} \int_0 \frac{(A\sin\omega\tau + \gamma W\tau)e^{\frac{W^2}{4k}\tau}}{(t-\tau)^{3/2}}\, e^{\frac{z^2}{4k(t-\tau)}}\, d\tau,$$

$$T = T_0 - \gamma z - \gamma Wt + T^*, \tag{21}$$

$$T(z,t) = T_0 - \gamma z - \gamma Wt + \frac{z e^{\frac{W}{2k}z - \frac{W^2}{4k}t}}{2\sqrt{k\pi}} \int_0 \frac{(A\sin\omega\tau + \gamma W\tau)e^{\frac{W^2}{4k}\tau}}{(t-\tau)^{3/2}}\, e^{\frac{z^2}{4k(t-\tau)}}\, d\tau, \tag{22}$$

FIELD EXPERIMENT

The data used in this research were collected at the Naqu flux site on the Tibetan Plateau during an intensive observation period from July 15 to 21, 1998. Details on the instruments and the various data processing techniques are provided at the web site: http://monsoon.t.u-tokyo.ac.jp/tibet/data/iop/pbltower/doc/naqu-fx.txt. This site was designed for the GAME/Tibet project at the position (91.54E, 31.22N) with an altitude of 4580 m. The ground surface covered by sparse short grass was flat and homogeneous. The soil at the site was predominantly sandy silt loam. Soil temperatures were measured at 0.5-h intervals at depths of 0.015 m and 0.04 m below surface by using two Campbell TCAV-averaging Soil Thermocouple Probes. It must be noted that the surface skin temperature was measured with a downward-facing radiometer (CNR1) at the height of 1.5 m. In this process, the Stefan-Boltzmann Law

$$I\uparrow = \varepsilon\sigma T_{0e}^{\,4}, \tag{23}$$

was used, where $I\uparrow$ is long wave (infrared, IR) radiation emitted upward from the surface, $\epsilon = 0.96$ is the infrared emissivity (Garratt, 1992, p. 292), $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant, and T_{0e} is the ground effective radiative temperature (in Kelvin). In addition, the volumetric soil water content was measured by two CS615 soil moisture reflectometers in the surface layer, that is, from the ground surface to a depth of 0.15 m. Figure 1a shows the variation in soil temperatures with time at three different layers at the Naqu site from July 15 to July 21, 1998. $T_{0.015}$ and $T_{0.04}$ are soil temperatures measured at depths of 0.015 m and 0.04 m. Figure 1b shows the averaged volumetric soil water content, and Fig. 1c gives the precipitation during this period. It is apparent that both soil temperature and volumetric soil water content responded dramatically to precipitation.

RESULTS AND DISCUSSION

Estimations of Soil Thermal Diffusivity k and Liquid Water Flux Density W

In our analysis above, we assumed the soil temperature varies sinusoidally. In reality, the soil temperature distribution depends on many factors, such as accepted radiation energy, cloud cover, vegetation cover, and some internal physical processes. We used a seven-point smoothing technique to deal with the soil temperatures (results are shown in Fig. 2). It must be noted that in the period selected, there are some gaps caused by precipitation.

The maximum temperature averaged over 7 days at the depth of 0.015 m reached 295.98 K, and at the depth of 0.04 m it reached 293.7 K. Therefore, we will use two different sine functions in our attempt to best approximate the curves of soil temperatures at these depths. These are $285 + 10.98\sin(t\pi/12 - x\pi)$ and $285 +$

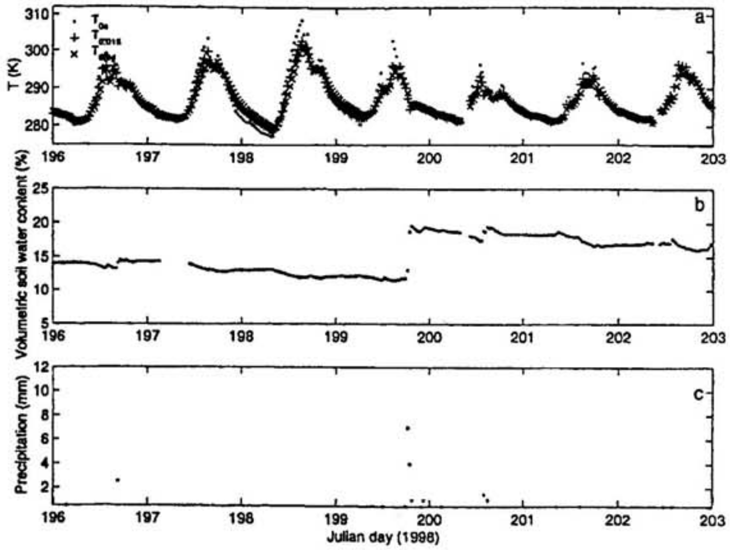

Fig. 1. Variation of soil temperature (T in Kelvin), volumetric water content (%) and precipitation (mm) with time at the Naqu site in the Tibetan Plateau from Julian 196 to 202, 1998.

$8.7\sin(t\pi/12 - y\pi)$, where t is time in hours, x and y are variables, 285 is the soil temperature at 0900 (local time) at 0.015 m (this is considered as the base of daily variation), 10.98 is the averaged amplitude of soil temperature at a depth of 0.015 m, and 8.7 is the averaged amplitude of soil temperature at a depth of 0.04 m. Results show that $x = 0.84$ and $y = 0.89$. We determined soil thermal diffusivity $k = 0.85 \times 10^{-6}\ m^2s^{-1}$ and liquid water flux density $W = 4.3 \times 10^{-6}\ m^{-3}\,s^{-1}\,m^{-2}$ by applying $A_1 = 8.7$ m, $A_2 = 10.98$ m, $\Phi_1 = -0.89\pi$, and $\Phi_2 = -0.84\pi$ to Eq. (17).

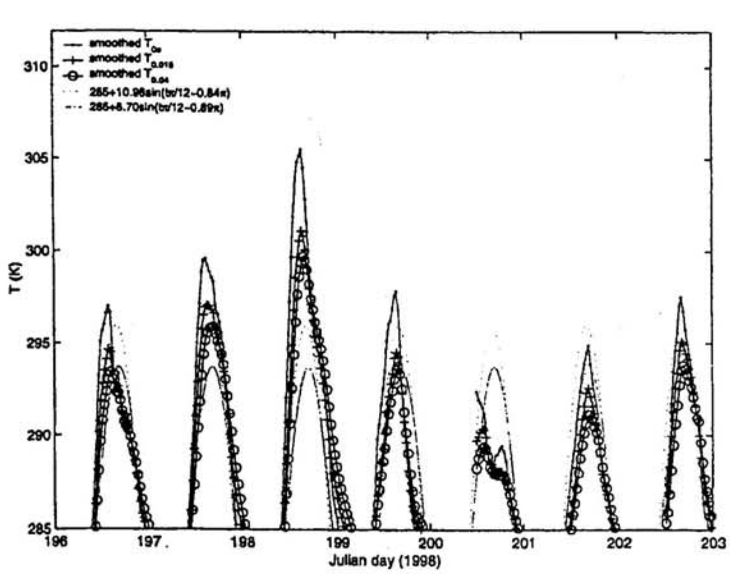

Fig. 2. Variation of smoothed soil temperature (T in Kelvin) and two sine curves with time at the Naqu site in the Tibetan Plateau from Julian 196 to 202, 1998.

Garratt (1992, p. 291) offered some representative values of thermal diffusivity. $k = 0.85 \times 10^{-6}\ m^2 s^{-1}$, obtained in this paper, is very close to the value in his table for thermal diffusivity of sand soil with volumetric soil water content of 20% ($0.84 \times 10^{-6}\ m^2 s^{-1}$). Ren et al. (2000) gave liquid soil water fluxes for sand, sandy loam, and clay loam. Their values ranged from $1.16 \times 10^{-5}\ m^3\ s^{-1}\ m^{-2}$ to $6.31 \times 10^{-5}\ m^3\ s^{-1}\ m^{-2}$. These are larger than our value ($W = 4.3 \times 10^{-6}\ m^3\ s^{-1}\ m^{-2}$) because their experimental work was done in water-saturated soil materials.

Modeling of Soil Temperature Profiles

Assuming $\gamma = 0$, we rewrite Eq. (22) as follows:

$$T(z,t) = T_0 + \frac{z e^{\frac{w}{2k}z - \frac{w^2}{4k}t}}{2\sqrt{k\pi}} \int_0 \frac{(A \sin\omega\tau) e^{\frac{w^2}{4k}\tau}}{(t-\tau)^{3/2}} e^{-\frac{z^2}{4k(t-\tau)}} d\tau, \tag{22'}$$

Because both soil thermal diffusivity k and liquid soil water flux W are determined, the profile of soil temperature can be obtained by using Eq. (16) or (22′) when surface temperature distribution is known. A 2-day period from Julian Day 208 to 209 in 1998 is chosen to test our equations. Figure 3 is the same as Fig. 1 except during this period. Results for Julian Day 208 to 210 are similar to those seen in Fig. 1 for Julian Days 196 to 203. No precipitation distribution is presented in Fig. 3 since there was no precipitation. Figure 4 shows the measured values together with our modeled values at the depth of 0.015 m. In this process, we fill the gaps of surface temperature by extrapolation and interpolation. These modeled values are from Eqs. (16) and (22′) for two cases: (i) $W = 0$ (the convective process is not considered) and (ii) $W = 4.3 \times 10^{-6}\ m^3\ s^{-1}\ m^{-2}$ (the convective process is considered). One finds that (i) both Eqs. (16) and (22′) give more realistic results when the convective process is considered, (ii) Eqs. (16) and (22) generate almost the same results for both cases mentioned above, and (iii) all modeled maximum (minimum) values are higher (lower) than measurement. We also modeled soil temperature at the depth of 0.04 m, but because the results are similar to those in Fig. 4, the figure is not shown here.

CONCLUSIONS

The soil thermal conduction-convection equation has been solved by applying the traditional methods: the harmonic method (HM) and the Laplace transform method (LTM). We concluded that if the assumption of steady-periodicity for soil temperature is valid, methods based on

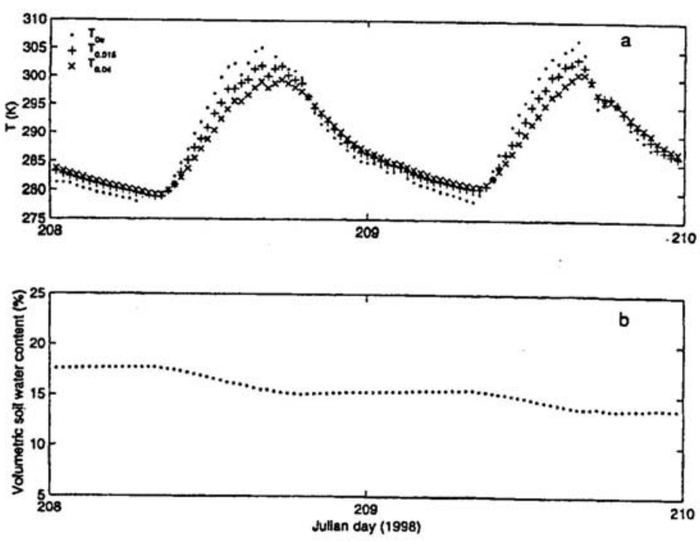

Fig. 3. Variation of soil temperature (T in Kelvin) and volumetric water content (%) with time at the Naqu site in the Tibetan Plateau from Julian 208 to 209, 1998.

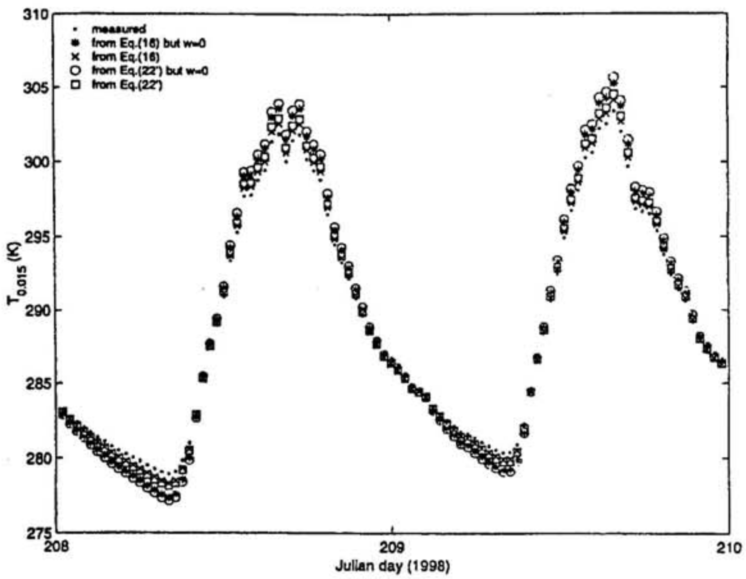

Fig. 4. Variation of the measured and modeled soil temperatures (T in Kelvin) at the depth of 0.015 m with time at the Naqu site in the Tibetan Plateau from Julian 208 to 209, 1998.

harmonic analysis and the Laplace transform can be used to test soil thermal diffusivity, even when the assumption of vertical homogeneity is not fully satisfied. The resolutions obtained by the two methods are expected to be applied in models. Because the HM is simpler to program, we recommend its use.

A simple method (Eq. (17)) to determine values of soil heat diffusivity and liquid soil water flux accurately is presented. With this method, we determined the soil thermal diffusivity $k = 0.85 \times 10^{-6}$ m^2 s^{-1} and the liquid water flux density $W = 4.3 \times 10^{-6}$ m^3 s^{-1} m^{-2} for the Naqu site.

An experimental evaluation of the proposed methods shows that our models, which take into account soil thermal convection processes, give more realistic results.

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