Image Transformation

Outline

- Geometric
- Photometric
- Application: Mosaicing

Geometric

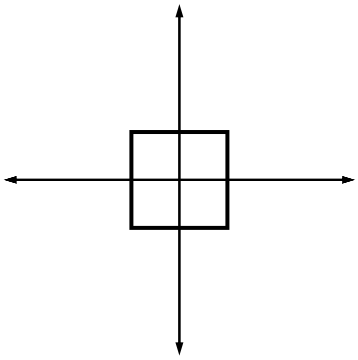
- Estimate 2D transformation

$$T(x, y) \rightarrow (x', y')$$

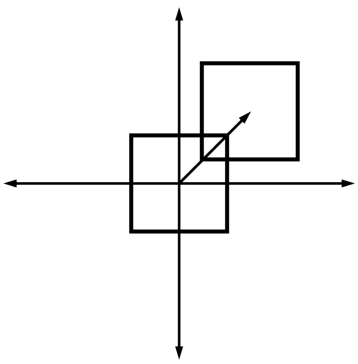
2-D Transformations

- Types
  - Translation
  - Scaling
  - Rotation
  - Shear

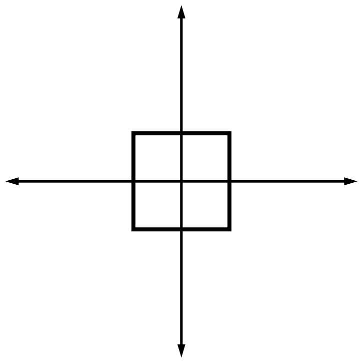
2-D Translation



2-D Translation

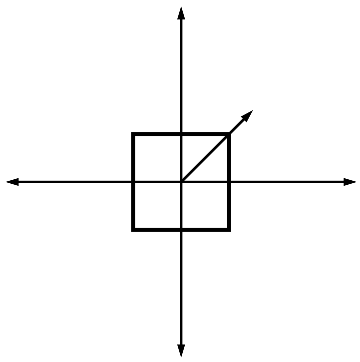


2-D Translation



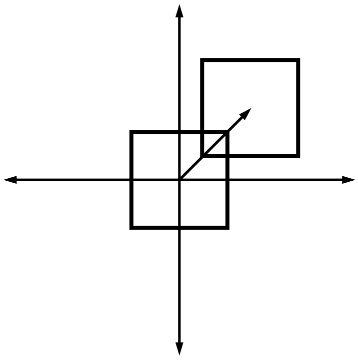
$$\begin{pmatrix} x \\ y \end{pmatrix}$$

2-D Translation



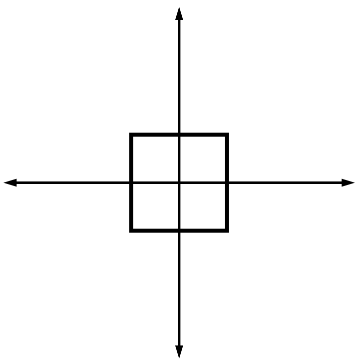
$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

2-D Translation

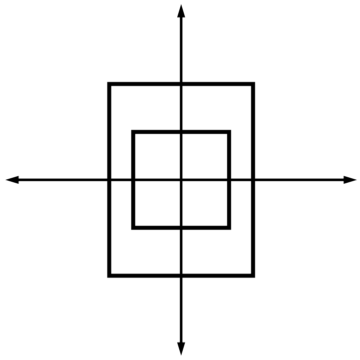


$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

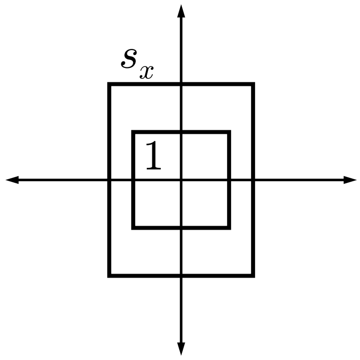
2-D Scaling



2-D Scaling

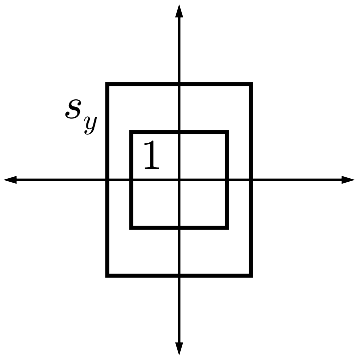


2-D Scaling



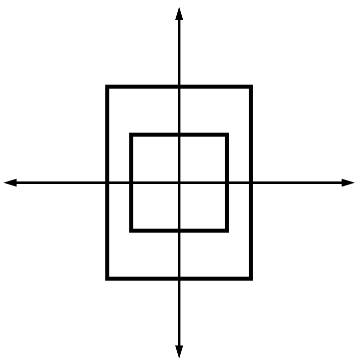
Horizontal shift proportional to horizontal position

2-D Scaling



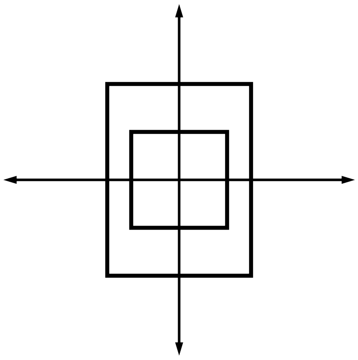
Vertical shift proportional to vertical position

2-D Scaling



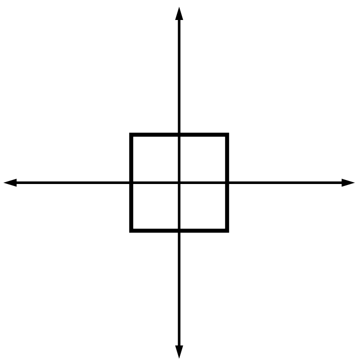
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x \cdot x \\ s_y \cdot y \end{pmatrix}$$

2-D Scaling

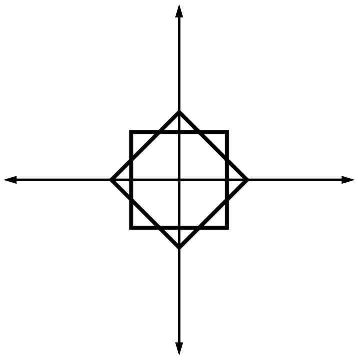


$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

2-D Rotation

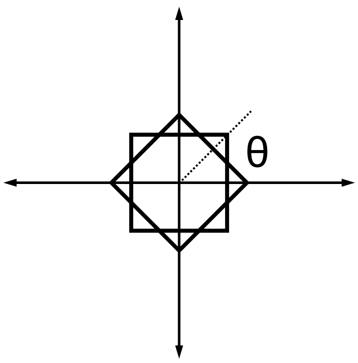


2-D Rotation



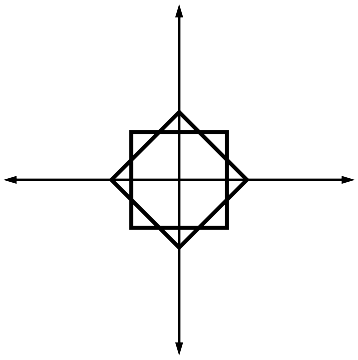
This is a counterclockwise rotation

2-D Rotation



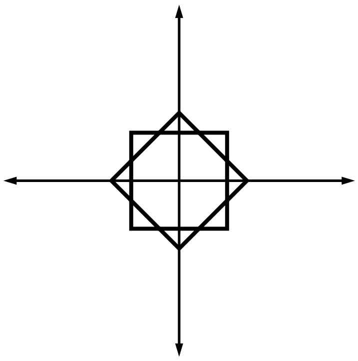
This is a counterclockwise rotation

2-D Rotation



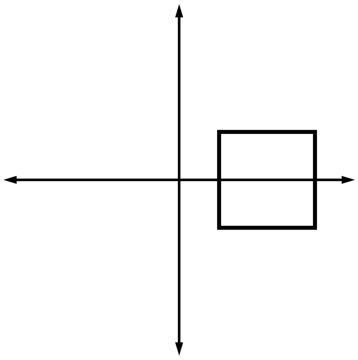
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix}$$

2-D Rotation



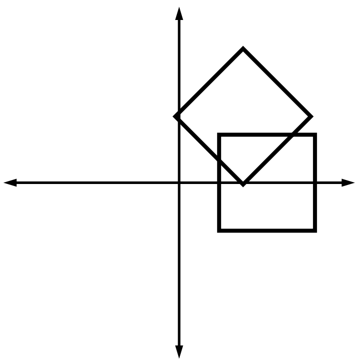
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

2-D Rotation (uncentered)



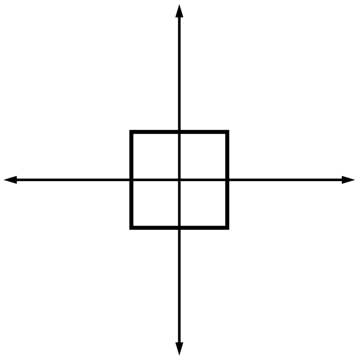
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

2-D Rotation (uncentered)

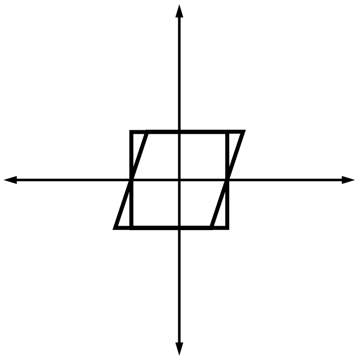


$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

2-D Shear (horizontal)

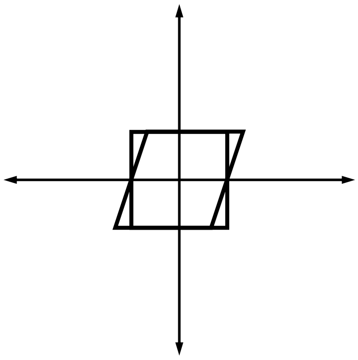


2-D Shear (horizontal)



Horizontal displacement proportional to vertical position

2-D Shear (horizontal)



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Representing Transformations

- Note that we've defined translation as a vector addition but rotation, scaling, etc. as matrix multiplications
- It's inconvenient to have two different operations (addition and multiplication) for different forms of transformation
- It would be desirable for all transformations to be expressed in a common form

Representing Transformations

- Note that we've defined translation as a vector addition but rotation, scaling, etc. as matrix multiplications
- It's inconvenient to have two different operations (addition and multiplication) for different forms of transformation
- It would be desirable for all transformations to be expressed in a common form
  - **Solution: Homogeneous coordinates**

Homogeneous Coordinates

- Let $\mathbf{x} = (x_1, x_2)^T$ be a point in 2D space
- Change to *homogeneous* coordinates:

$$\mathbf{x} \Rightarrow (\mathbf{x}^T, 1)^T$$

- Defined up to scale

$$(\mathbf{x}^T, 1)^T \Rightarrow (w\mathbf{x}^T, w)^T$$

- Can go back to non-homogeneous representation as follows:

$$(\mathbf{x}^T, w)^T \Rightarrow \mathbf{x}/w$$

Homogeneous Coordinates: Translations

- 2-D translation of a point was expressed as a vector addition $\mathbf{x}' = \mathbf{x} + \mathbf{t}$
- Homogeneous coordinates allow it to be written as a multiplication by a 3 x 3 matrix:

$$\mathbf{x}' = \begin{pmatrix} \mathbf{Id} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{pmatrix} \mathbf{x}$$

Example: Translation with homogeneous coordinates

- Old way: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$
- New way:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Homogeneous Coordinates: Rotations, etc.

- A 2-D rotation, scaling, shear or other transformation normally expressed by a 2 x 2 matrix $\mathbf{R}$ is written in homogeneous coordinates with the following 3 x 3 matrix:

$$\mathbf{x}' = \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{pmatrix} \mathbf{x}$$

General Formulation

- General formulation

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 \\ t_7 & t_8 & t_9 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{aligned} x' &= \frac{t_1 x + t_2 y + t_3}{t_7 x + t_8 y + t_9} \\ y' &= \frac{t_4 x + t_5 y + t_6}{t_7 x + t_8 y + t_9} \end{aligned}$$

Compute the Transformation

$$\begin{aligned}(t_7x + t_8y + t_9)x' &= t_1x + t_2y + t_3\\(t_7x + t_8y + t_9)y' &= t_4x + t_5y + t_6\end{aligned}$$

$$\begin{pmatrix} x & y & 1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & x & y & 1 & xy' & yy' & y' \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \\ \mathrm{t}_9 \end{pmatrix} = \mathbf{0}$$

Compute the Transformation

When there are n corresponding points available, we have a linear system:

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & x_1x_1' & y_1x_1' & x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & x_1y_1' & y_1y_1' & y_1' \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ x_i & y_i & 1 & 0 & 0 & 0 & x_ix_i' & y_ix_i' & x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & x_iy_i' & y_iy_i' & y_i' \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & x_nx_n' & y_nx_n' & x_n' \\ 0 & 0 & 0 & x_n & y_n & 1 & x_ny_n' & y_ny_n' & y_n' \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \\ t_9 \end{pmatrix} = \mathbf{0}$$

Compute the Transformation

Simplified representation

$$Mu = \mathbf{0}$$

Find the transformation unknowns by SVD decomposition of the correlation matrix, and optimal u is the orthornomal axis associated with smallest singular value.

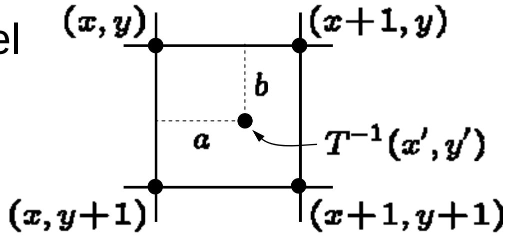
$$\min_u \|Mu\|^2 = \min_u (u^T M^T M u)$$

Photometric

- How to compute new pixel values when non-integral $T^{-1}(x', y')$
  - Nearest neighbor: Value of closest pixel
  - Bilinear interpolation (2 x 2 neighborhood)
  - Bicubic interpolation (4 x 4)

Bilinear Interpolation

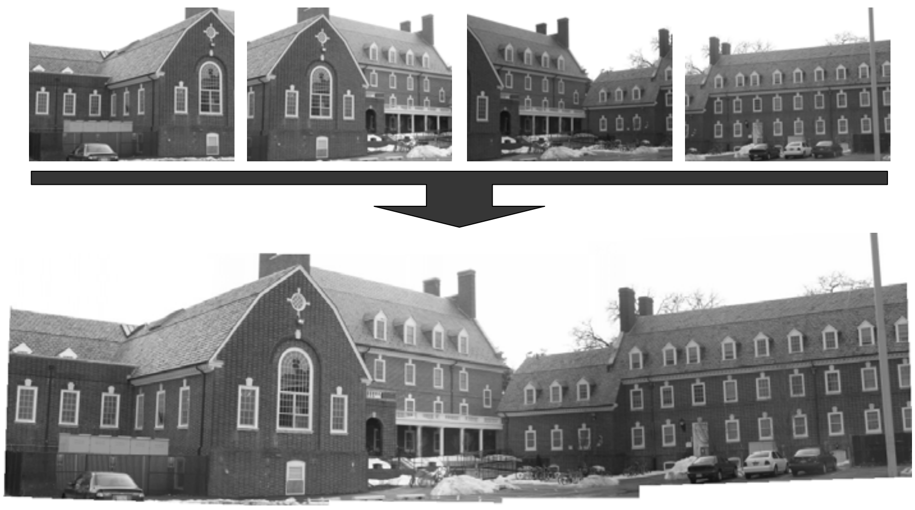
- Idea: Blend four pixel values surrounding source, weighted by nearness



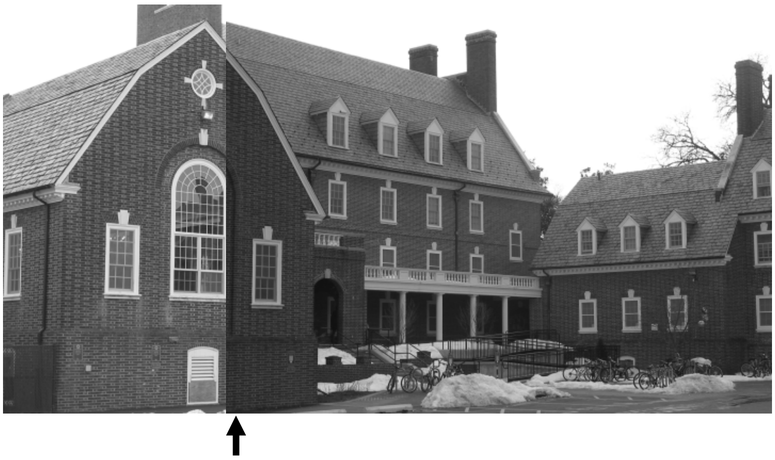
$$\mathbf{I}(x', y') = (1-b, b) \begin{bmatrix} \mathbf{I}(x,y) & \mathbf{I}(x+1,y) \\ \mathbf{I}(x,y+1) & \mathbf{I}(x+1,y+1) \end{bmatrix} \begin{pmatrix} 1-a \\ a \end{pmatrix}$$

Vertical blend   Horizontal blend

Mosaicing



Geometric & Photometric "Seams"



Can we get rid of the visible line?