Hidden Surface Elimination: BSP trees

Outline

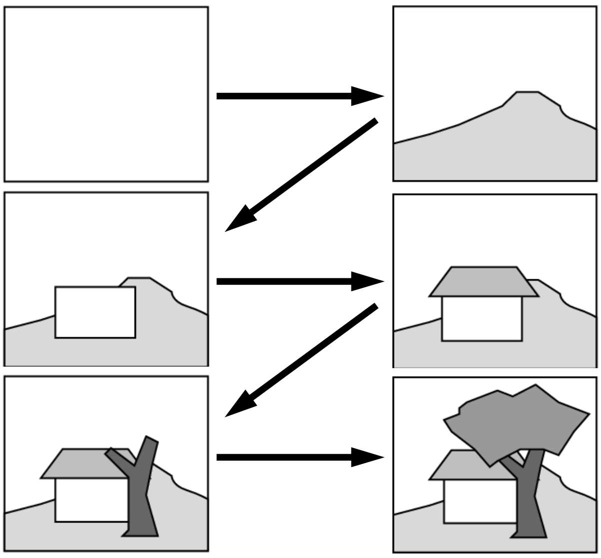
- Binary space partition (BSP) trees
  - Polygon-aligned

BSP Trees

- Basic idea: Preprocess geometric primitives in scene to build a spatial data structure such that tests from **any viewpoint** can be easily calculated later
- Examples of tests
  - **Visibility** for painter's algorithms
  - **Intersection testing** for ray tracing
- Generalization of binary search trees (1-D) to higher dimensions

Painter's algorithm

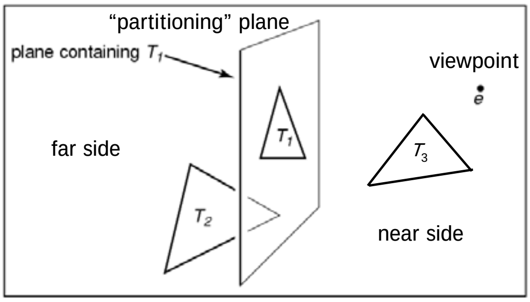
Draw primitives from back to front to avoid need for depth comparisons



from Shirley

BSP trees: Key property

- "Spatial sorting" keeps track of which side of lines/planes primitives are on
  - Objects on the same side as the viewer can be **drawn on top** of objects on the opposite side
  - Objects on one side **cannot intersect** objects on the other side

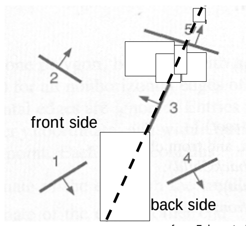


adapted from Shirley

  - "Polygon-aligned" means partitioning plane is always coplanar with a scene polygon, as opposed to arbitrarily positioned

Building 2-D "line-aligned" BSP trees

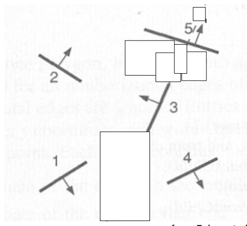
- Pick oriented line segment (i.e., has a normal) from list as the root
- Rest of lines partitioned according to which side they are on
  - "Partitioning" line placed at root of subtree
  - Sets of lines on "front" side and "back" side correspond to left & right subtrees, respectively
- Recurse on each child



from Foley *et al.*

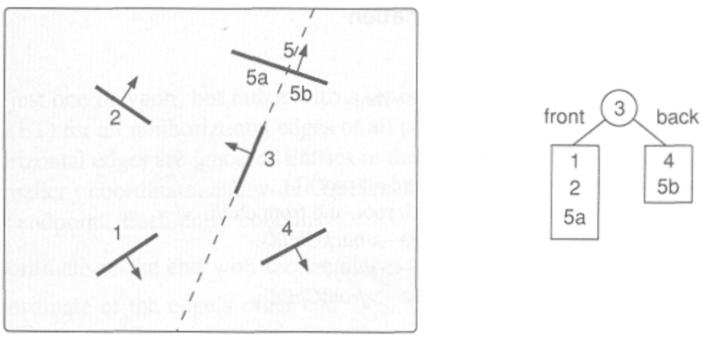
Building 2-D BSP trees: Issues

- How to pick line with which to partition
- What to do with lines that cross partitioning line
  - Split them (standard)
  - Or: Put a copy on each side of boundary
- When to stop recursing
  - n or fewer primitives per leaf ($n = 1$ is standard)
  - Or: Threshold on recursion depth



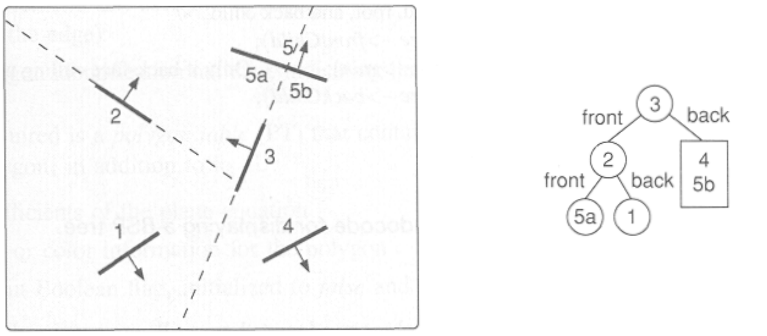
from Foley *et al.*

2-D BSP tree: Building example



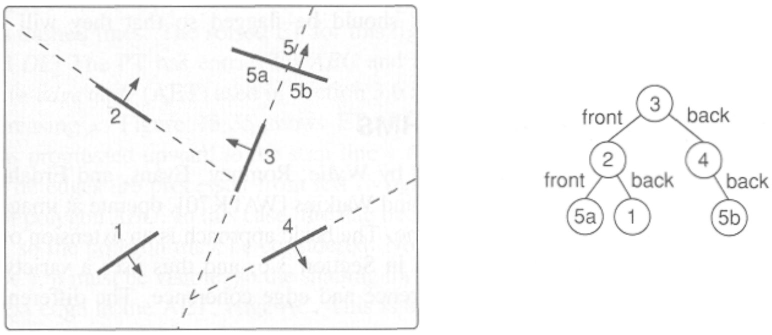
from Foley *et al.*

2-D BSP tree: Building example



from Foley *et al.*

2-D BSP tree: Building example



from Foley *et al.*

Building 2-D BSP trees: Pseudocode

adapted from Foley *et al.*

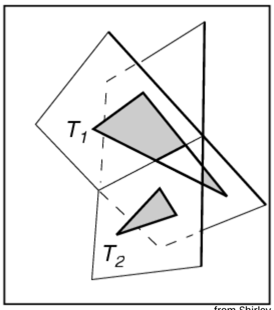
```
BSP_tree *BSP_makeTree(line *lineList)
{
   line root, l, backPart, frontPart, *backList, *frontList;

   if (lineList == NULL)
        return NULL;
   else {
        root = BSP_selectAndRemoveLine(lineList);
        backList = frontList = NULL;
        for (each line l in lineList) {
                if (l in front of root)
                        BSP_addToList(l, frontList);
                else if (l in back of root)
                        BSP_addToList(l, backList);
                else {
                        BSP_splitLine(l, root, frontPart, backPart);
                        BSP_addToList(frontPart, frontList);
                        BSP_addToList(backPart, backList);
   }    }       }
   return BSP_mergeTree(BSP_makeTree(frontList), root, BSP_makeTree(backList));
}
```

Picking partitioning lines: Criteria

- **Painter's algorithm**
  - Every object must be drawn $\rightarrow$ Entire tree is traversed
  - So overall tree size should be as small as possible
  - ➢ Minimize splitting
- **Ray tracing** (later in course...)
  - Several paths from root to leaves traversed looking for intersections
  - So tree <u>depth</u> more important than overall size
  - ➢ Balance primitives on each side

Partitioning lines: "Least-crossed" heuristic

- For painter's algorithm, we want to choose partitioning lines that minimize the number of splits
- A particular line can be tested to see how many lines cross it and therefore would have to be split if it were the partitioning line
- Good procedure in practice:
  1. Randomly select a small number of candidate partitioning lines (e.g., 5-10 out of 1,000)
  2. Calculate number of lines that cross each candidate
  3. Use candidate with least crossings as next partition

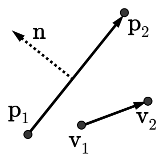


from Shirley

Crossings for two planes

Building 2-D BSP trees: Details

- How to parametrize partitioning line $\mathbf{l}$ from line segment $\mathbf{p}_1\mathbf{p}_2$?
  - Homogeneous line form $\mathbf{l} = (a,\, b,\, c)^T$ for segment is same as line it's on
  - We can obtain $\mathbf{l}$ from $\mathbf{p}_1$, $\mathbf{p}_2$ via $\mathbf{l} = \mathbf{p}_1 \times \mathbf{p}_2$
  - Normal vector of line $\mathbf{l}$ is $\mathbf{n} = (a,\, b)^T$



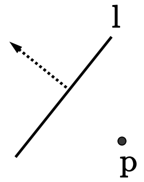
- How to decide which side of partitioning line $\mathbf{l}$ a line segment $\mathbf{v}_1\mathbf{v}_2$ is on?
  - If both points $\mathbf{v}_1$, $\mathbf{v}_2$ are on the front side of $\mathbf{l}$, line $\mathbf{v}_1\mathbf{v}_2$ is on front side
  - If both $\mathbf{v}_1$, $\mathbf{v}_2$ are on the back side, $\mathbf{v}_1\mathbf{v}_2$ is on back side
  - If $\mathbf{v}_1$, $\mathbf{v}_2$ are on different sides, $\mathbf{v}_1\mathbf{v}_2$ crosses partitioning line $\mathbf{l}$

Building 2-D BSP trees: Details

- How to split crossing lines?
  - Find intersection point $\mathbf{x}$ of $\mathbf{v}_1\mathbf{v}_2$ with $\mathbf{l}$
    - Let $\mathbf{l}'$ be homogeneous form of line defined by $\mathbf{v}_1\mathbf{v}_2$
    - By definition, we want a point $\mathbf{x}$ that is on both lines $\mathbf{l}$ and $\mathbf{l}'$. This would imply that $\mathbf{l} \cdot \mathbf{x} = \mathbf{l}' \cdot \mathbf{x} = 0$
    - Just looking at these as vectors, a dot product of 0 means that $\mathbf{x}$ is orthogonal to both $\mathbf{l}$ and $\mathbf{l}'$
    - Because **cross product** is orthogonal to both multiplicands, $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$ satisfies this requirement and thus defines the point of intersection
  - Given intersection $\mathbf{x}$:
    - If $\mathbf{v}_1$ is on front side of $\mathbf{l}$: Output $\mathbf{v}_1\,\mathbf{x}$ as front part and $\mathbf{x}\mathbf{v}_2$ as back part
    - If $\mathbf{v}_2$ is on front side of $\mathbf{l}$: Output $\mathbf{x}\mathbf{v}_2$ as front part and $\mathbf{v}_1\,\mathbf{x}$ as back part

Painter's algorithm: Tree traversal

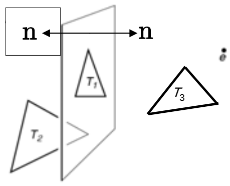
- Want farthest-to-nearest ordering of primitives for painter's algorithm
  - If view location is on **front side** of a partitioning line:
    - Lines on back side are farther
    - Lines on front side are nearer
  - If view location is on **back side** of a partitioning line:
    - Lines on front side are farther
    - Lines on back side are nearer



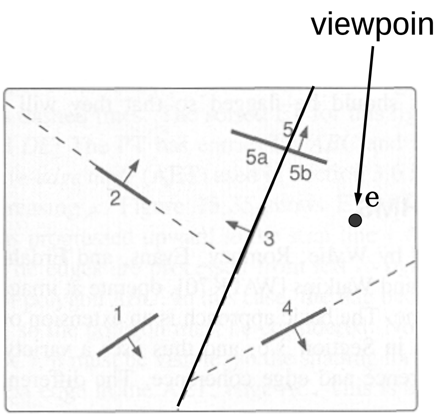
- Which side of a partitioning line $\mathbf{l}$ is a point $\mathbf{p}$ on?
  - Assuming $\mathbf{l}$, $\mathbf{p}$ in homogeneous form, use homogeneous line test $F(\mathbf{p}) = \mathbf{l} \cdot \mathbf{p}$
    - $F > 0 \Rightarrow \mathbf{p}$ is on **back side** of $\mathbf{l}$
    - $F < 0 \Rightarrow \mathbf{p}$ is on **front side** of $\mathbf{l}$
    - $F = 0 \Rightarrow \mathbf{p}$ is on line $\mathbf{l}$ (arbitrarily treat as back side)

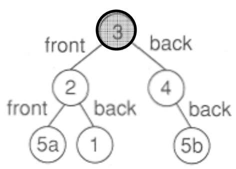
Painter's algorithm 2-D BSP tree traversal: Pseudocode

```
void BSP_displayTree(BSP_tree *tree, point viewLocation)
{
    if (tree != NULL) {
       if (viewLocation is in front of tree->root) {

           // Display back child, root, then front child

          BSP_displayTree(tree->backChild, viewLocation);
          displayLine(tree->root);
          BSP_displayTree(tree->frontChild, viewLocation);
       }
       else {

           // Display front child, root, then back child

          BSP_displayTree(tree->frontChild, viewLocation);
          displayLine(tree->root);    // back-facing line—can cull by skipping
          BSP_displayTree(tree->backChild, viewLocation);
       }
}
```



Painter's algorithm: Example BSP tree traversal

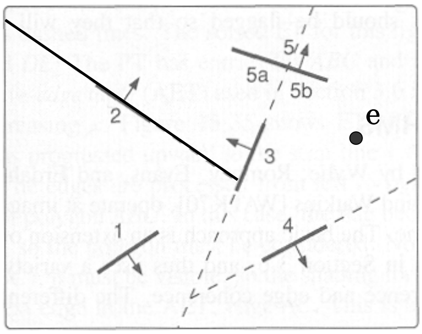


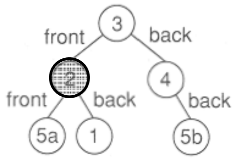


from Foley *et al.*

Behind root (node 3), so display everything in front of (left subtree = nodes 1, 2, 5a), then root (node 3), then everything behind (right subtree = nodes 4 and 5b)

Painter’s algorithm: Example BSP tree traversal

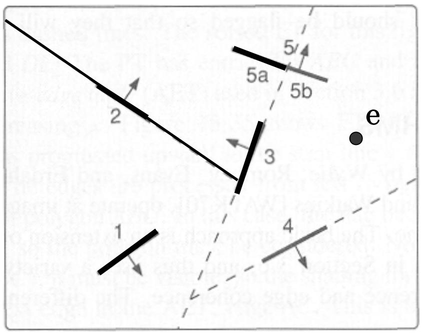


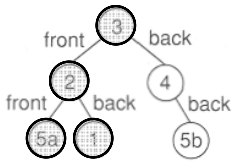


from Foley *et al.*

In front of root (node 2), so display everything behind (right subtree = node 1),
then root (node 2), then everything in front of (left subtree = node 5a)

Painter’s algorithm: Example BSP tree traversal





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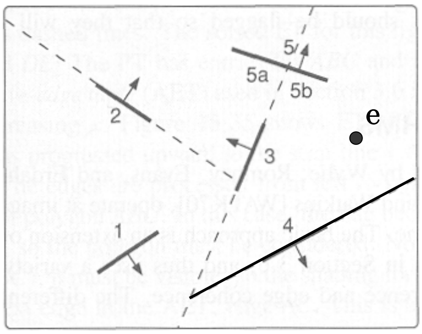
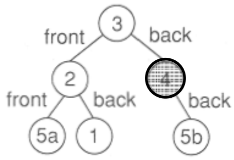
In front of root (node 2), so display everything behind (right subtree = node 1),
then root (node 2), then everything in front of (left subtree = node 5a)

Painter's algorithm: Example BSP tree traversal

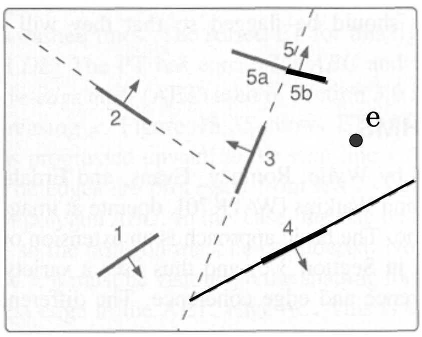


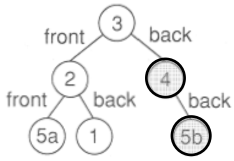


from Foley *et al.*

Behind root (node 4), so display everything in front of (left subtree = NULL), then root (node 4), then everything behind (right subtree = node 5b)

Painter's algorithm: Example BSP tree traversal



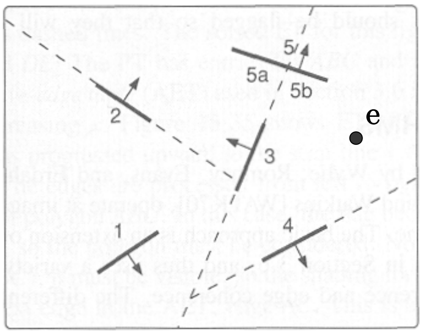


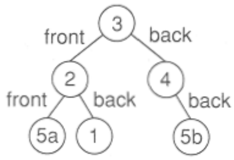
from Foley *et al.*

Behind root (node 4), so display everything in front of (left subtree = NULL), then root (node 4), then everything behind (right subtree = node 5b)

Painter's algorithm: Example BSP tree traversal

Final order: 1, 2, 5a, 3, 4, 5b



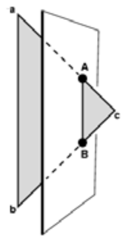


from Foley *et al.*

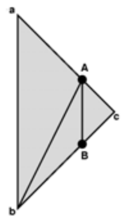
Every node is visited from back-to-front, so this is an $O(n)$ operation (n is the number of primitives **after** splitting)

3-D BSP Trees

- Analog of 2-D method, but now we are talking about 3-D polygon primitives and partitioning planes
- What's different about this vs. lines?
  - Must parametrize plane from polygon
  - Point-plane "sidedness" test is analogous to point-line test
    - Just use homogeneous form of plane equation
  - Line (each edge of polygon)-plane intersection instead of line-line intersection
  - Polygon splitting instead of line splitting



Triangle crossing partitioning plane

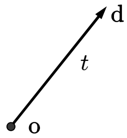


Triangle splitting

from Shirley

3-D BSP Trees: Details

- Parametrize plane from polygon
  - Cross product of edges gets plane normal $\mathbf{n} = (a, b, c)^T$
  - Solve for d from single point on plane $(x, y, z)^T$ via plane equation $ax + by + cz + d = 0$
- Line-plane intersection
  - Unfortunately, there is no simple homogeneous form for lines in 3-D like in 2-D
  - Instead:
    - Express point $\mathbf{p}$ on a ray as some distance t along direction $\mathbf{d}$ from origin $\mathbf{o}$: $\mathbf{p} = \mathbf{o} + t\mathbf{d}$
    - Use plane equation $\mathbf{n} \cdot \mathbf{x} + d = 0$, substitute $\mathbf{o} + t\mathbf{d}$ for $\mathbf{x}$, and solve for t
    - Then plug t back into $\mathbf{p} = \mathbf{o} + t\mathbf{d}$ to get $\mathbf{p}$



BSP Trees: Notes

- Works best for static scenes
  - Moving primitives can cross partitioning lines
  - Dynamic adjustment of tree possible, but slows things down