**Sinusoidal Waves: Amplitude, Period & Frequency**

Math 117  
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**Isu # 07c**  
COHH 4135

**Article 00: Readings.** Pleasurable and invigorating reading assignments follow. Please partake of the intrinsic joy therein found.

Rdng [pp 173 - 177] - - - - - - - - - - - - - - - - - Exer[p 180] :: #01-26.

**Article 01: Sinusoidal Wave Concepts.** Consider the graph of \( y = \sin(t) \) or of \( y = \cos(t) \) as displayed in Fig (A) & Fig (B), below.

![FIG_01A](image1) ![FIG_01B](image2)

Such graphs as these are called *sinusoidal wave graphs*. The name, of course, refers to the shape of the *sine function graph*. However, we have already noted that the *sine & cosine graphs* have the same shape. The only difference is the PLACEMENT of the *vertical, y-axis*! Also, note that in the above wave graphs the "wave crests" are closer together in Fig (A) than those crests in Fig (B). Hence, the *cycle length* of the first is SHORTER than that of the second. The (least) *cycle length* is called the (fundamental) *PERIOD* of the function wave. Hence, the (A)-wave has a SMALLER PERIOD than the (B)-wave. The period is the (shortest) length on the \( t \)-axis which will SPAN ONE FULL CYCLE. The period or cycle length is also called the *WAVE-LENGTH*. Another feature of *sinusoidal waves* is the *HEIGHT of the CRESTS* (from the horizontal axis). This feature of the wave is called the *AMPLITUDE* of the function wave.

**Article 02: Sinusoidal Wave Features.** The graphs of \( y = \sin(t) \) or of \( y = \cos(t) \) are both *PERIODIC* with *period* = \( 2\pi \) and each has *AMPLITUDE* = 1.

**(2.1) AMPLITUDE.** In order to create *amplitude* = \( A \), simply MULTIPLY by \( A \) : \( y = A \cdot \sin(t) \) and \( y = A \cdot \cos(t) \). Now, the *maximum value* of either function will be \( A \) ; and, the *least value* of either will be \( (-A) \). Hence, the *AMPLITUDE* will simply be the *numerical coefficient* of the sinusoidal function.

**(2.2) PERIOD (or Wave-Length).** Recall that the *periods* of \( y = \sin(t) \) or of \( y = \cos(t) \) are \( 2\pi \). Hence, ONE COMPLETE CYCLE of each SPANS a \( t \)-axis length of \( 2\pi \). Now, imagine the function \( f(t) = \sin(bt) \), for some real number \( b \in \mathbb{R} \). We ask what number \( p \) is the *period* of this function \( f(t) \)? We inspect: \( f(t + p) = f(t) \) in order to answer this question. Since, \( \sin(t) \) has *period* = \( 2\pi \), then: \( b(t + p) = bt + 2\pi \Rightarrow \sin(bt + p) = \sin(bt + 2\pi) \Rightarrow f(t + p) = f(t) \). So,... solving for \( p \), we find that: \( p = \frac{2\pi}{b} \). CHECKING: \( f(t + p) = \sin b \left(t + \frac{2\pi}{b}\right) = \sin(bt + 2\pi) = \sin(bt) = f(t) \).
(2.3) FREQUENCY. The FREQUENCY of a sinusoidal wave is simply the NUMBER of Wave-Lengths (or Period Cycles) that will fit into a UNIT length on the t-axis. Hence, if TWO wave cycles will fit into a unit length, then the frequency is \( f = \frac{1}{2} \). Alternatively, if only ONE-THIRD of a wave length will fit into a unit length, then the frequency is \( f = \frac{1}{3} \). Given this DESCRIPTION of frequency, it is PLAIN TO SEE that the numerical relation between period = \( p \) AND frequency = \( f \) is given by
\[
f = \frac{1}{p}, \text{ or } p = \frac{1}{f}, \text{ or } fp = 1.
\]

Article 03: Fundamental Observations & Procedural Illustrations.

(3.1) \( y = \sin(t) \) and \( y = \cos(t) \) have amplitude = 1 and period = \( 2\pi \).

(3.2) \( y = A \cdot \sin(bt) \) and \( y = A \cdot \cos(bt) \) have amplitude = \( A \) and period = \( \frac{2\pi}{b} \).

(3.3) \( y = A \cdot \sin(bt + c) \) and \( y = A \cdot \cos(bt + c) \) have amplitude = \( A \) and period = \( \frac{2\pi}{b} \).

Note that: \( \sin(bt + c) = \sin\left(b\left(t + \frac{c}{b}\right)\right) \). Now, let \( \tau = \left(t + \frac{c}{b}\right) \) so that it follows \( \sin\left(b\left(t + \frac{c}{b}\right)\right) = \sin(\tau) \). Hence, since \( \sin(\tau) \) has period = \( \frac{2\pi}{b} \), then so does \( \sin(bt + c) \).

(3.4) In order to SKETCH the graph of: \( y_t = \sin(bt + c) \), proceed as follows:

(a) First, sketch the graph of: \( y = \sin(\tau) \), since this will provide a wave with period = \( \frac{2\pi}{b} \) and such that: \( y_\tau = 0 \) at \( \tau = 0 \).

(b) Now write: \( y_t = \sin(bt + c) = \sin\left(b\left(t + \frac{c}{b}\right)\right) \). Then, observe that \( t = 0 \) at \( \tau = \frac{c}{b} \).

So, at \( \tau = \frac{c}{b} \), INSERT the vertical \( y_t \)-axis. The graph of \( y_t = \sin(bt + c) \) is done!

(3.5) In order to SKETCH the graph of: \( y_t = A \cdot \sin(bt + c) \), SKETCH as directed in Item (3.4) AND provide the wave with an AMPLITUDE = \( A \).

(3.6) In order to SKETCH the graph of: \( y_t = A \cdot \cos(bt + c) \), SKETCH using PARALLEL procedures to those used for the SINE function.

Article 04: Concept Examiners. Develop complete & detailed solutions for the following morsels of intellectual pleasure!

01. Determine the AMPLITUDE and PERIOD and FREQUENCY of each sinusoidal function below:
   (a) \( 5 \sin(2\pi t) \) (b) \( \frac{5}{8} \cdot \cos(6t) \) (c) \( \frac{\pi}{4} \cdot \sin\left(\frac{1}{2}t + \pi\right) \) (d) \( \cos\left(8\pi t - \frac{1}{2}\pi\right) \) (e) \( \frac{1}{2} \cdot \cos\left(\frac{7}{2}\pi\right) \)

02. SKETCH each of the sinusoidal functions appearing in Exercise 01, above.

ANS(01): (a) 5 1 1 (b) \( \frac{5}{8} \frac{5}{8} \frac{5}{8} \frac{5}{8} \) (c) 0 4\pi \( \frac{1}{2}\pi \) (d) 1 1 \( \frac{1}{4} \pi \) 4 (e) \( \frac{1}{2} \) \( \frac{2\pi^2}{1} \) \( \frac{1}{2}\pi \)