Return and Risk: The Capital-Asset Pricing Model (CAPM)

Expected Returns (Single assets & Portfolios), Variance, Diversification, Efficient Set, Market Portfolio, and CAPM

Expected Returns and Variances

- **For Individual Assets**
- Calculations based on *Expectations* of future;

$$E(R) = \Sigma\ (p_s \times R_s)$$

- *Variance (or Standard Deviation)*:
  - a measure of variability;
  - a measure of the amount by which the returns might deviate from the average (E(R))

$$\sigma^2 = \Sigma\ \{p_s \times [R_s - E(R)]^2\}$$

Covariance

- *Covariance:* **Co (joint) Variance** of two asset's returns
  - a measure of variability
- Cov(AB) will be large & '+' if :
  - 'A' & 'B' have large Std. Deviations and/or
  - 'A' & 'B' tend to move together
- Cov(AB) will be '-' if:
  - Returns for 'A' & 'B' tend to move *counter* to each other

Correlation Coefficient

- *Correlation Coefficient:*
  - **Standardized Measure of the co-movement between two variables**

$\rho_{AB} = \sigma_{AB} / (\sigma_A \sigma_B)$; I.e., $Cov(AB)/\sigma_A \sigma_B$;

  - same sign as covariance
  - Always between (& including) -1.0 and + 1.0

Portfolio Expected Returns

- *Portfolio*:
  - a collection of securities (stocks, etc.)
- *Portfolio Expected Returns*:
  - Weighted sum of the expected returns of individual securities

$$E(R_p) = X_A E(R)_A + X_B E(R)_B$$

Portfolio Variance

- *Portfolio Variance:*
  - **NOT** the weighted sum of the individual security variances
  - **Depends on the interactive risk . I.e.,**
  - Correlation between the returns of individual securities
  - $\sigma_P^2 = X_A^2\sigma_A^2 + 2\ X_A\ X_B\ \sigma_{AB} + X_B^2\sigma_B^2$

    $\sigma_{AB} = \rho_{AB}\ \sigma_A\sigma_B$

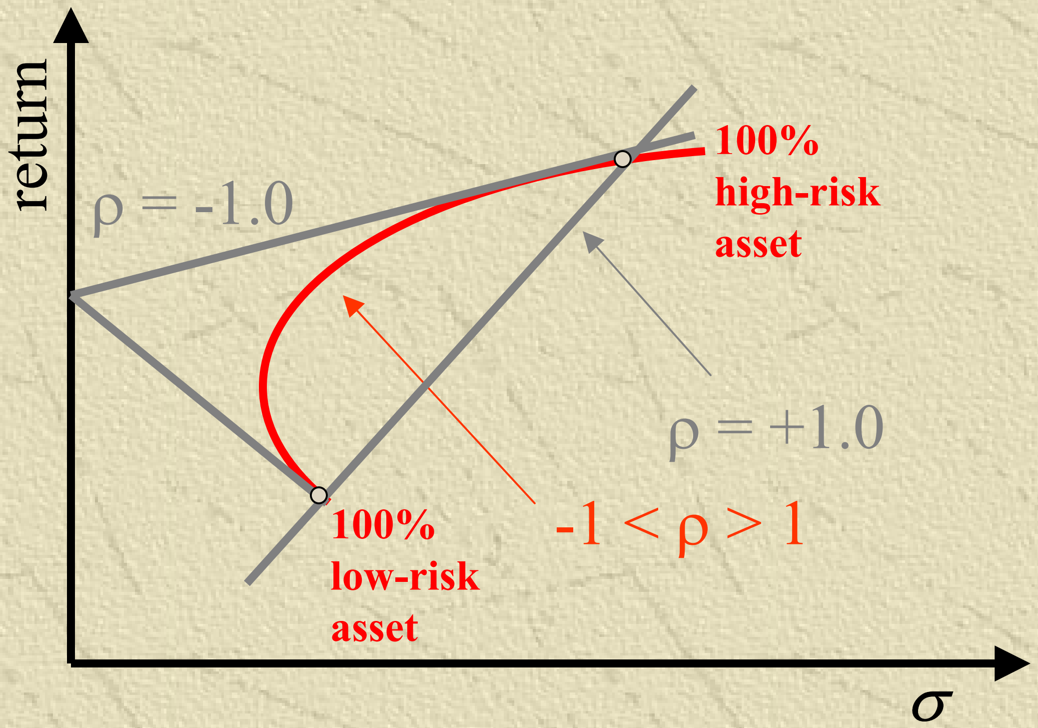
Diversification

- *Diversification Effect:*
  - <u>Actual</u> portfolio variance $\leq$ weighted sum of individual security variances
  - more pronounced when ρ is negative

Opportunity and Efficient Sets

- *Opportunity Set:*
  - Attainable or Feasible set of portfolios
    - constructed with different mixes of ‘A’ & ‘B’
- Are all portfolios in the Opportunity Set equally good? NO!
  - Only the portfolios on **Efficient Set**
    - Portfolios on the Efficient Set **dominate** all other portfolios
- What is a Minimum Variance Portfolio?

Efficient Sets and Diversification (2 security portfolios)



Portfolio Risk/Return Two Securities: Correlation Effects

- Relationship depends on correlation coefficient
- $-1.0 \leq \rho \leq +1.0$
- The smaller the correlation, the greater the risk reduction potential
- If $\rho = +1.0$, no risk reduction is possible

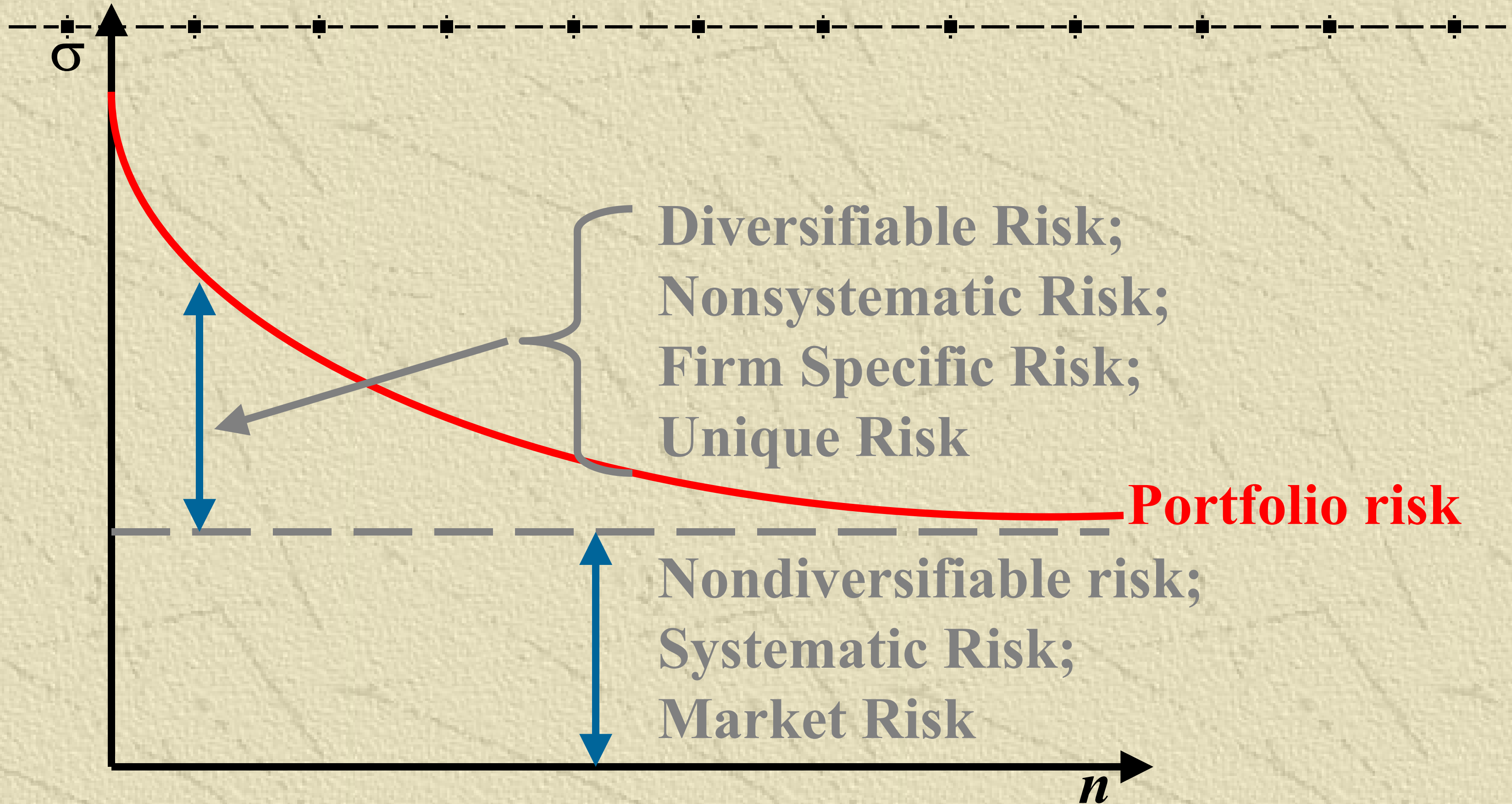
Efficient Sets (Continued)

- Efficient set with many securities
  - Computational nightmare!
  - Inputs required: ‘N’ expected returns, ‘N’ variances, $(N^2 - N)/2$ covariances.

Portfolio Diversification

- Investors are risk-averse
  - Demand $\Uparrow$ returns for taking $\Uparrow$ risk
- *Principle of Diversification*
  - Combining **imperfectly related** assets can produce a portfolio with less variability than a "typical" asset

Portfolio Risk as a Function of the Number of Stocks in the Portfolio

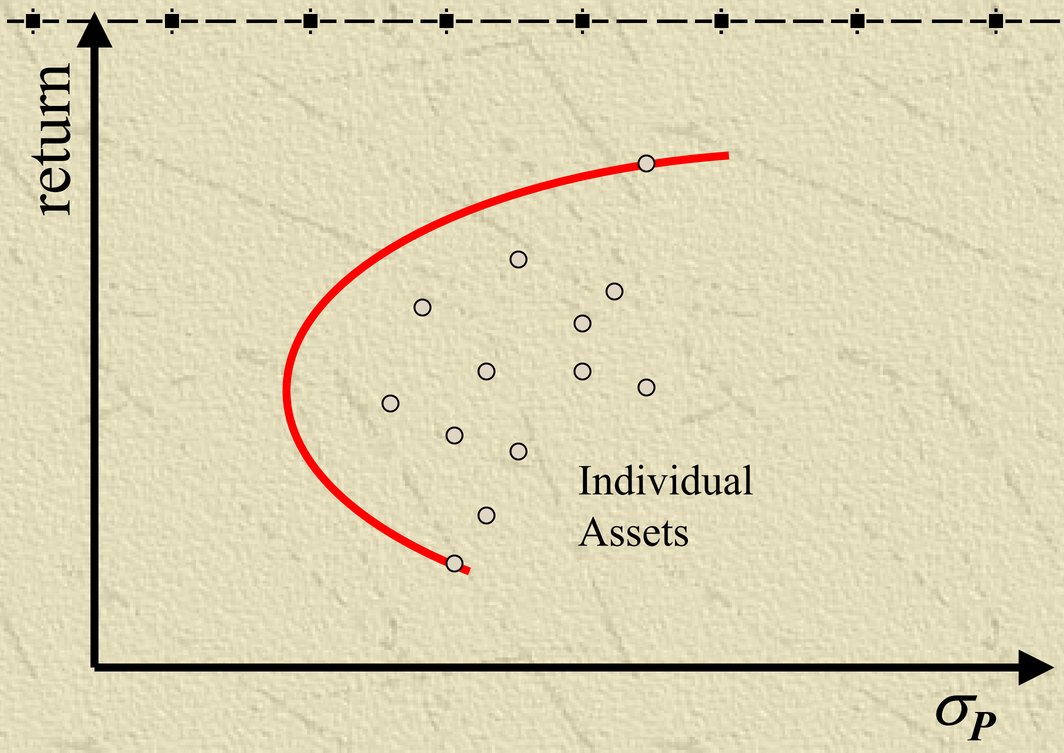


Thus diversification can eliminate some, but not all of the risk of individual securities.

Different Types of Risks

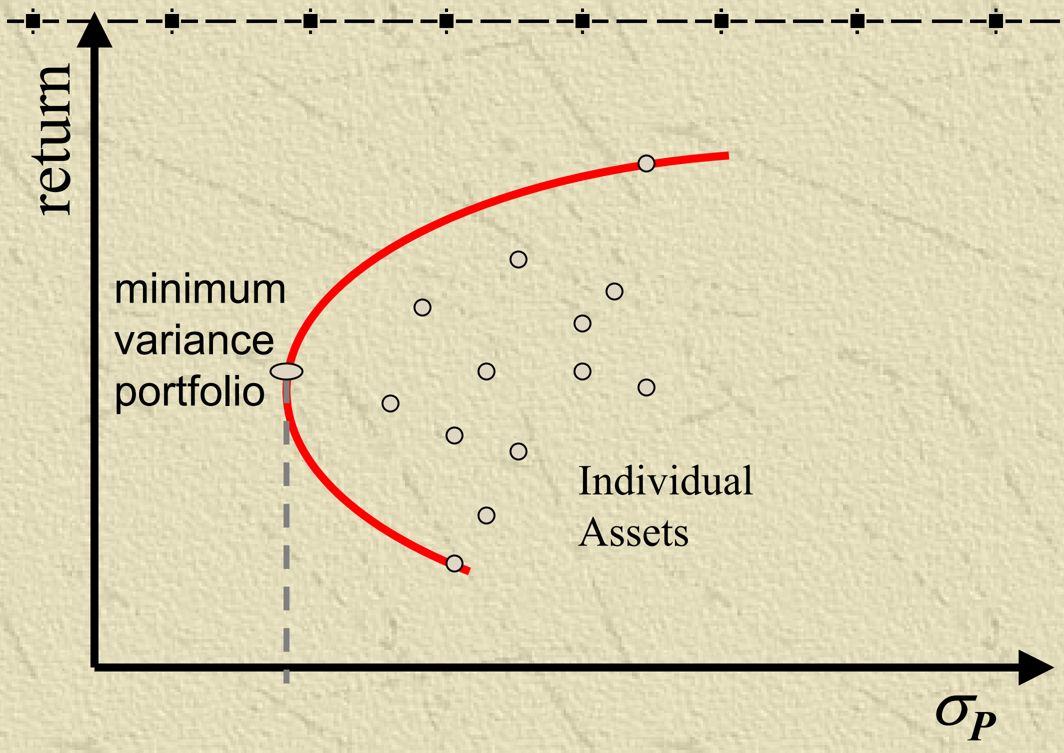
- Total risk of an asset:
  - Measured by σ or σ^2
- Diversifiable risk of an asset:
  - Portion of risk that is eliminated in a portfolio; *(Unsystematic risk)*
- Undiversifiable risk of an asset:
  - Portion of risk that is **NOT** eliminated in a portfolio; *(Systematic risk)*

The Efficient Set for Many Securities



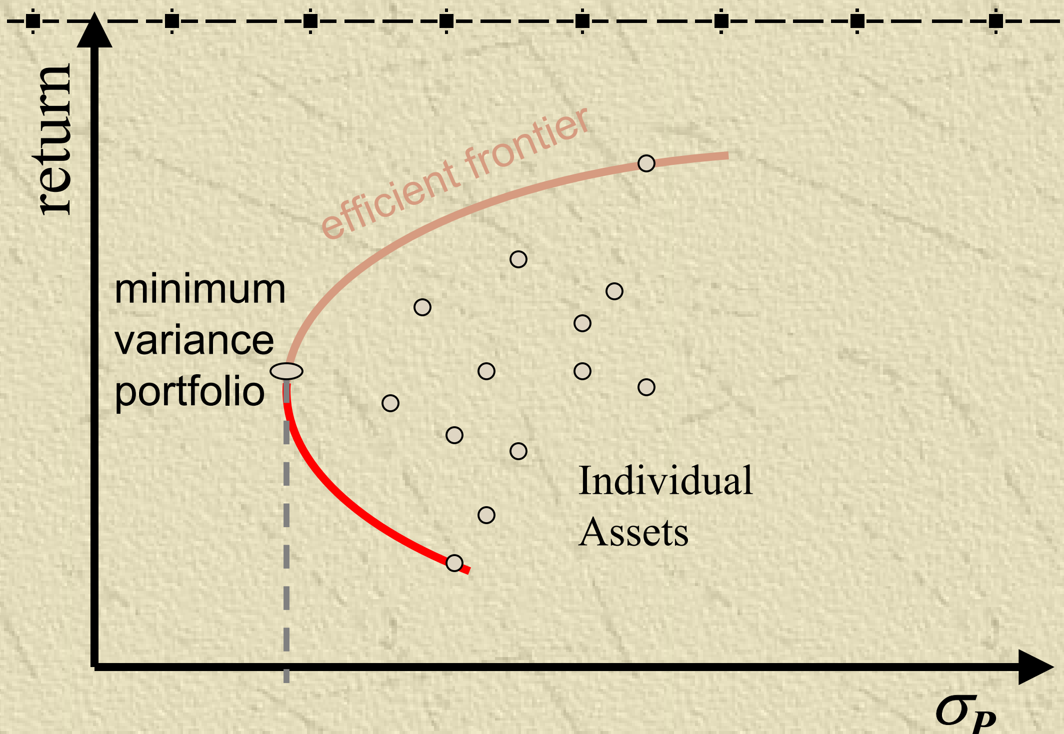
Consider a world with many risky assets; we can still identify the *opportunity set* of risk-return combinations of various portfolios.

The Efficient Set for Many Securities



Given the *opportunity set* we can identify the **minimum variance portfolio**.

10.5 The Efficient Set for Many Securities



The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

Efficient set in the presence of **riskless** borrowing/lending

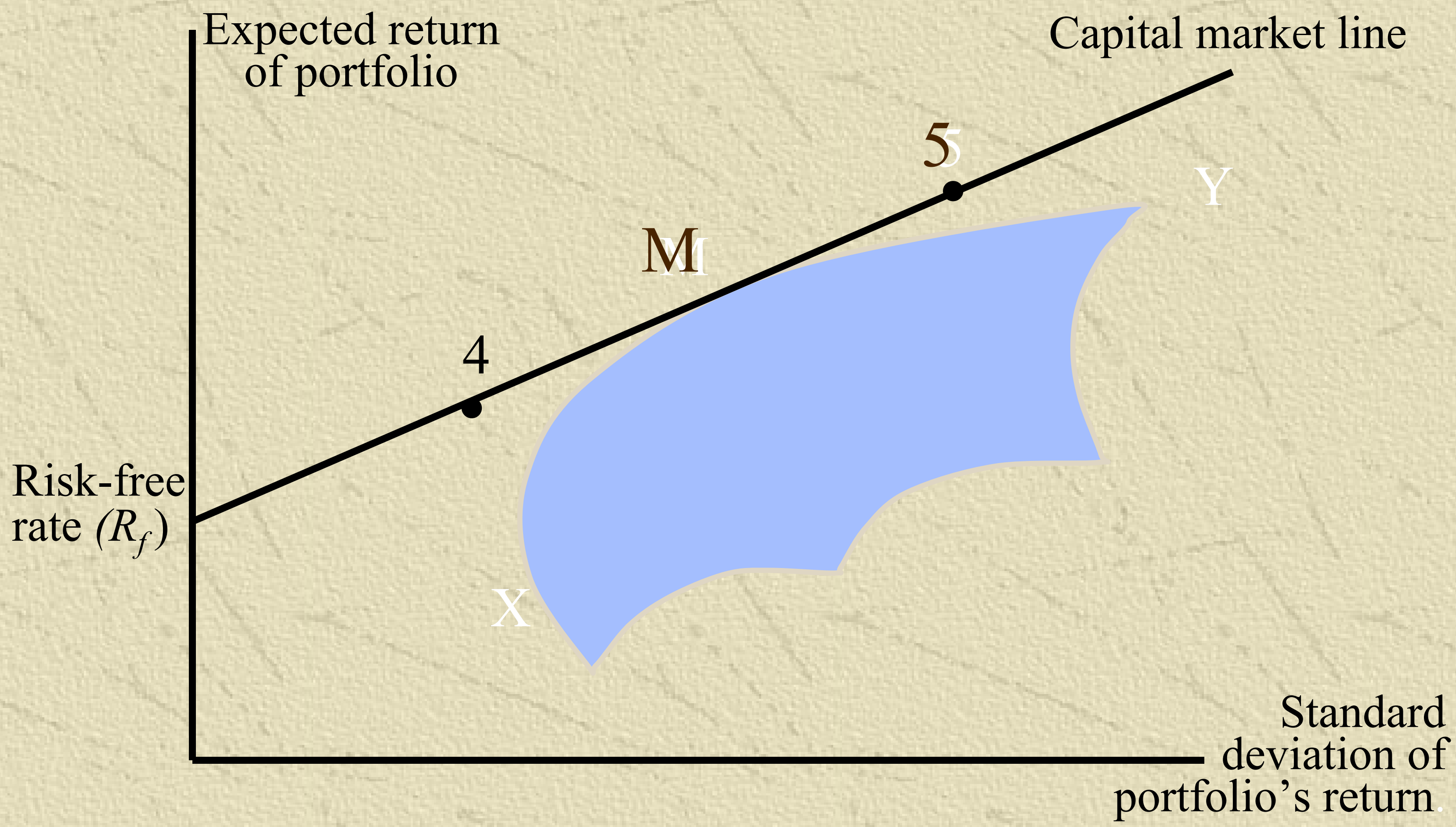
- A Portfolio of a risky and a riskless asset:

$E(R)_p = X_{risky} * E(R)_{risky} + X_{riskless} * E(R)_{riskless}$

$S.D._p = X_{riskless} * \sigma_{riskless}$

- Opportunity & Efficient set with 'N' risky securities and 1 riskless asset
  - tangent **line** from the riskless asset to the curved efficient set

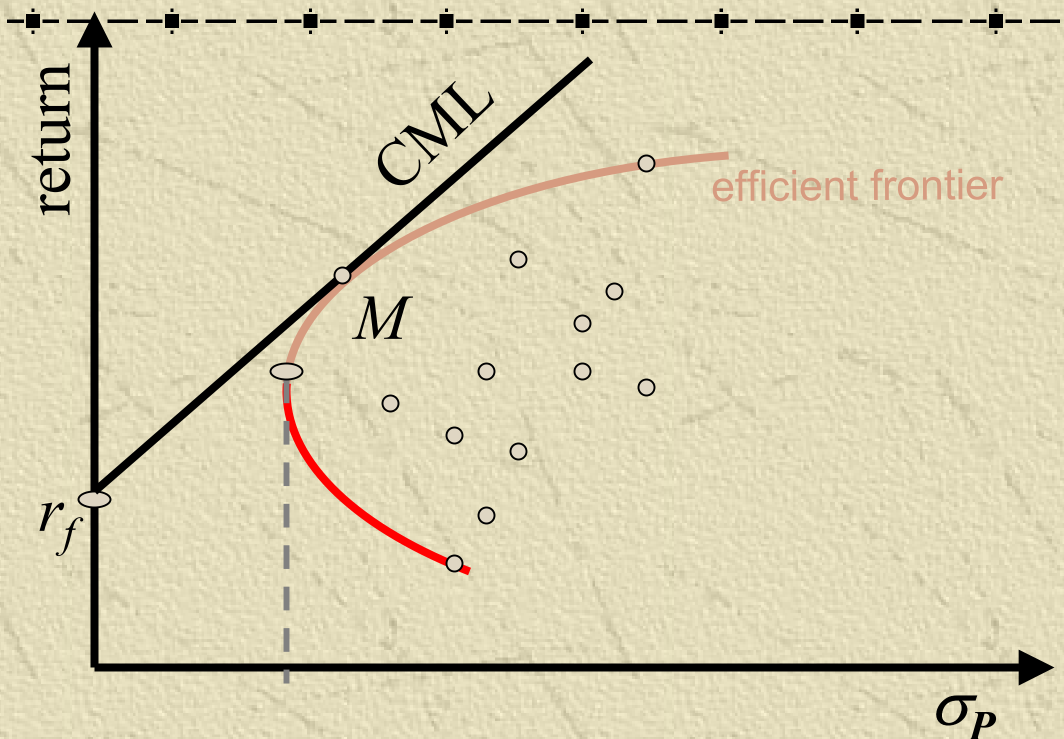
Capital Market Line



Efficient set in the presence of **riskless** borrowing/lending

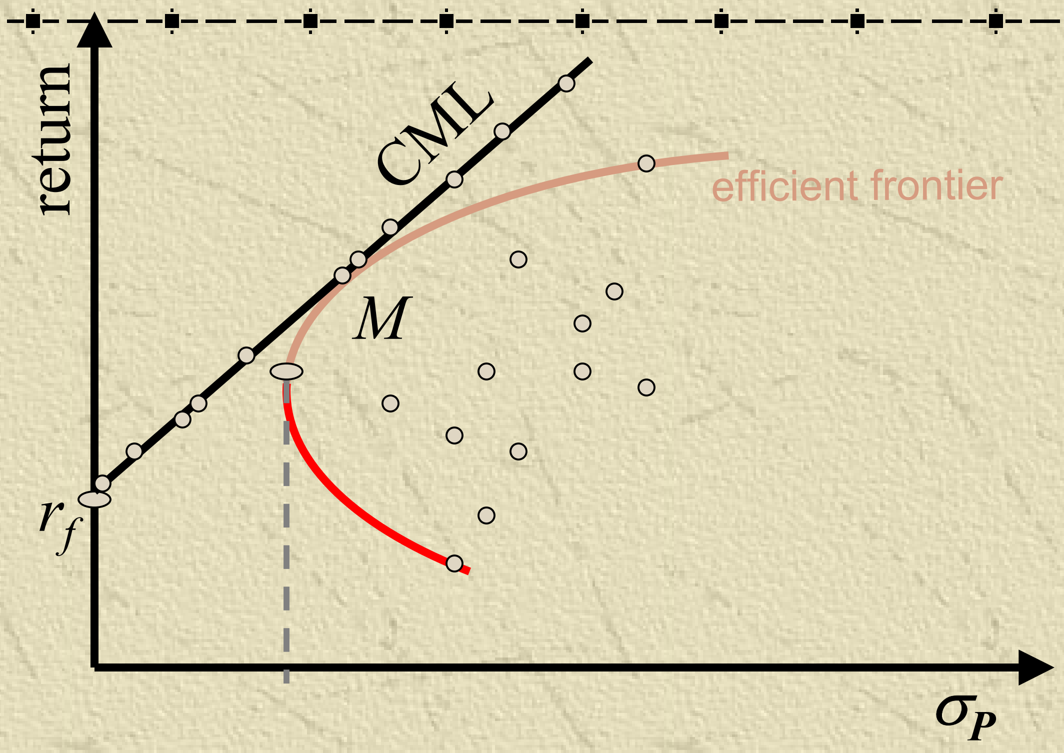
- *Capital Market Line*
  - efficient set of risky & riskless assets
  - investors' choice of the "optimal" portfolio is a function of their risk-aversion
- Separation Principle: investors make investment decisions in 2 separate steps:
  1. **All** investors invest in the same risky "asset"
  2. Determine proportion invested in the 2 assets?

The Separation Property



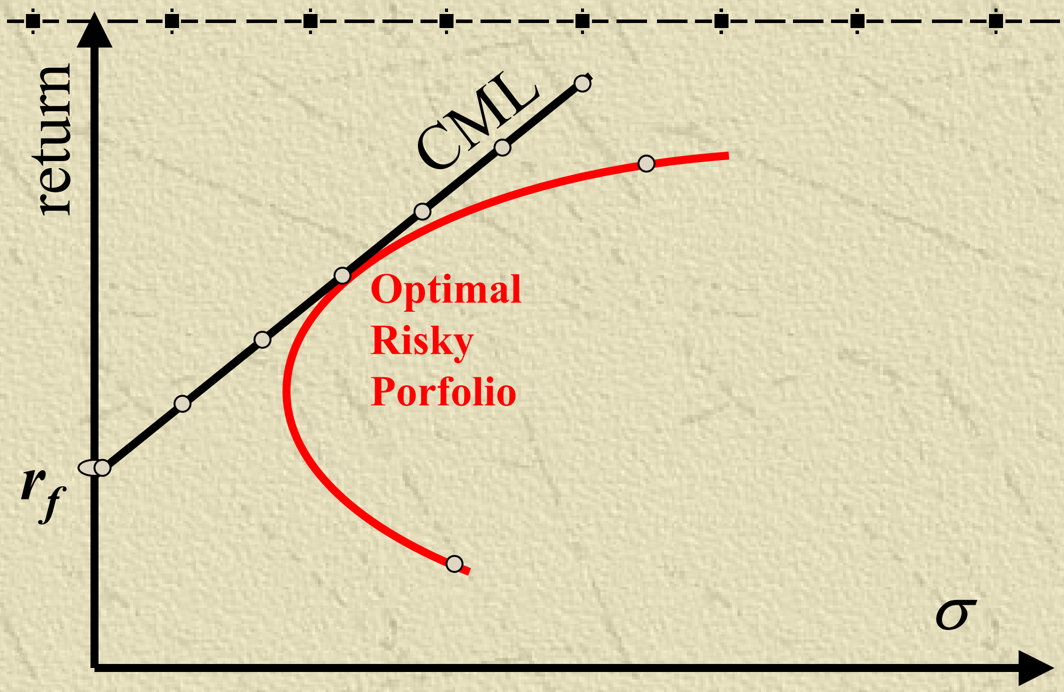
The Separation Property states that the market portfolio, *M*, is the same for all investors—they can *separate* their risk aversion from their choice of the market portfolio.

The Separation Property



Investor risk aversion is revealed in their choice of where to stay along the capital allocation line—not in their choice of the line.

The Separation Property



The separation property implies that portfolio choice can be separated into two tasks: (1) determine the optimal risky portfolio, and (2) selecting a point on the CML.

Market Equilibrium

- Homogeneous expectations
  - all investors choose the *SAME* risky **(Market)** portfolio and the same riskless asset.
    - Though different weights
  - Market portfolio is a w*ell-diversified portfolio*
- What is the *"Relevant"* risk of an asset?
  - The contribution the asset makes to the risk of the "market portfolio"
  - NOT the total risk (I.e., not σ or $\sigma^{2)}$

Definition of Risk When Investors Hold the Market Portfolio

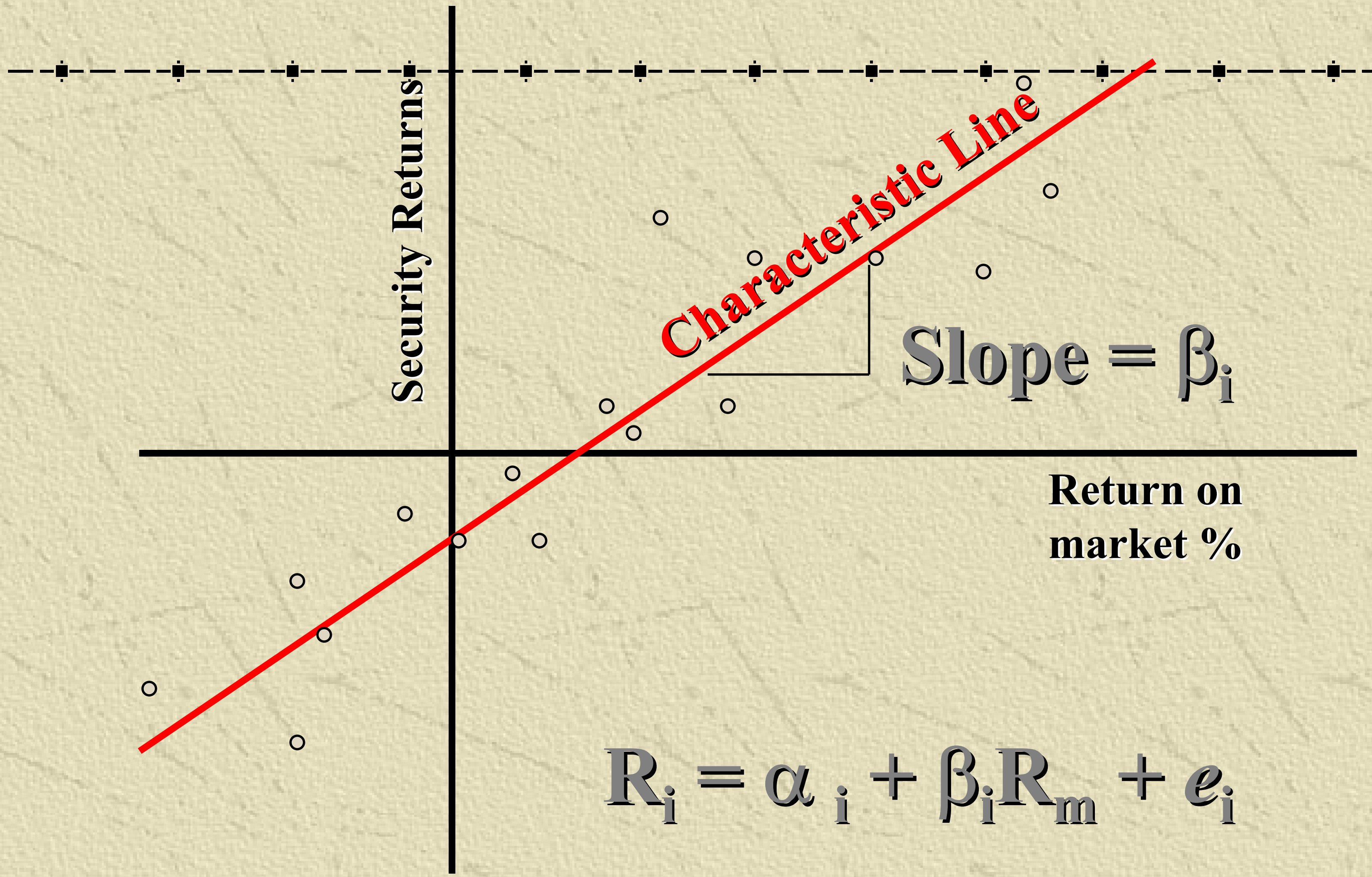
- **Beta**
- Beta measures the responsiveness of a security to movements in the market portfolio.

$$\beta_i = \frac{Cov(R_{i,} R_M)}{\sigma^2(R_M)}$$

Beta

- BETA
  - measures *only* the interactive (with the market) risk of the asset (systematic risk)
    - Remaining (unsystematic) risk is diversifiable
    - Slope of the characteristic line
- $\text{Beta}_{\text{portfolio}}$ = weighted average beta of individual securities
- β_m = average beta across ALL securities = 1

Estimating β with regression



$$R_i = \alpha_i + \beta_i R_m + e_i$$

Risk & Expected Returns (CAPM & SML)

- as risk ↑, you can *expect* return ↑ too
  - & vice-versa: As return ↑ , so does risk ↑
- Which Risk??
- *Systematic Risk Principle:*
  - Market only rewards investors for taking systematic (**NOT total**) risk
- WHY?
- Unsystematic risk can be diversified away

Relationship between Risk and Expected Return (CAPM)

- Expected Return on the Market:

$$\overline{R}_M = R_F + \text{Market Risk Premium}$$

Thus, Mkt. RP = $(R_M - R_F)$

- Expected return on an individual security:

$$\overline{R}_i = R_F + \beta_i \times \underbrace{(\overline{R}_M - R_F)}_{\text{Market Risk Premium}}$$

This applies to individual securities held within well-diversified portfolios.

Expected Return on an Individual Security

- This formula is called the Capital Asset Pricing Model (CAPM)

$$\bar{R}_i = R_F + \beta_i \times (\bar{R}_M - R_F)$$

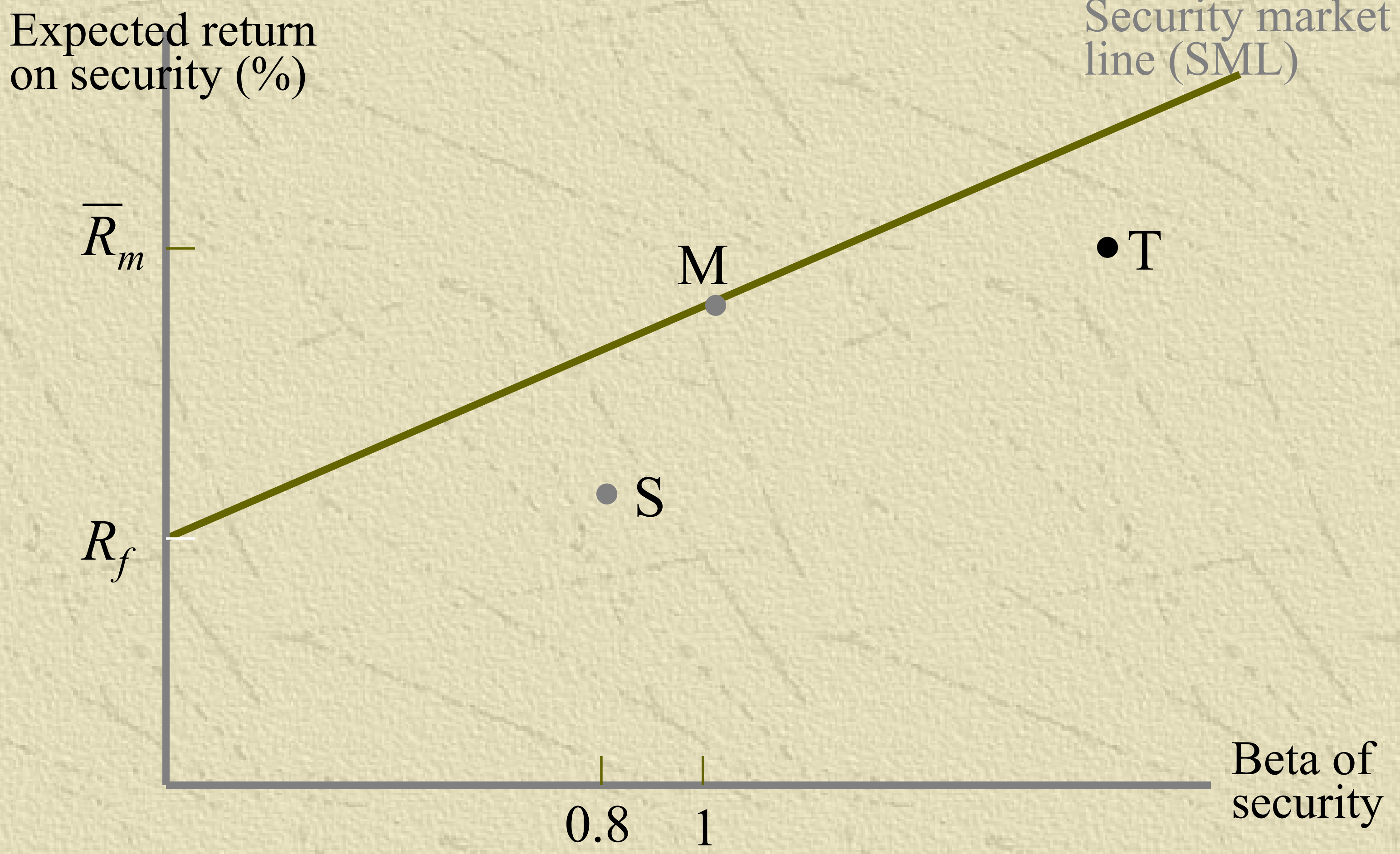
Expected return on a security	=	Risk-free rate	+	Beta of the security	×	Market risk premium

- Assume $\beta_i = 0$, then the expected return is R_F.
- Assume $\beta_i = 1$, then $\bar{R}_i = \bar{R}_M$

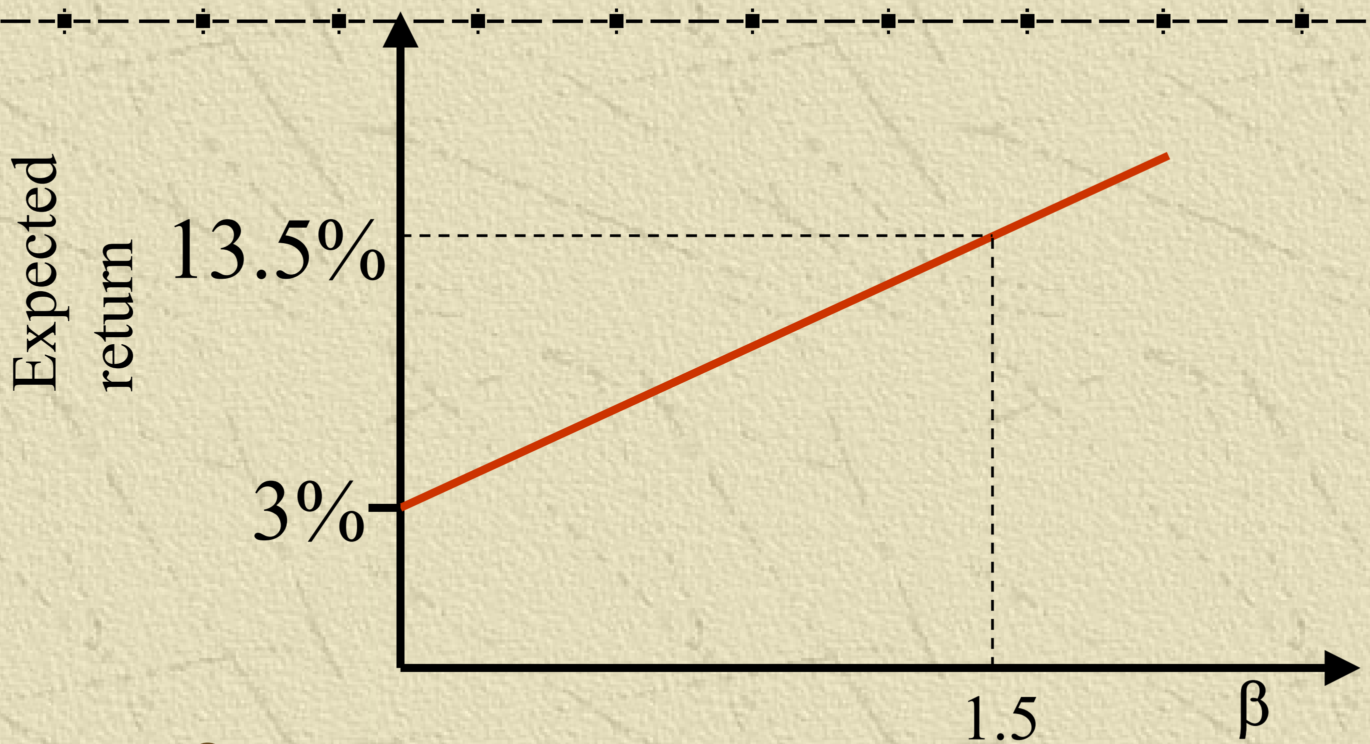
CAPM & SML-- Continued

- *SML:* graph between Betas and E(R)
- Salient features of SML:
  - Positive slope: As betas $\Uparrow$ so do E(R)
  - Intercept = R_F ; Slope = Mkt. RP
  - Securities that plot **below** the line are **Overvalued** and <u>vice-versa</u>

Security Market Line



Relationship Between Risk & Expected Return



$$\beta_i = 1.5 \qquad R_F = 3\% \qquad \overline{R}_M = 10\%$$

$$\overline{R}_i = 3\% + 1.5 \times (10\% - 3\%) = 13.5\%$$

CAPM & SML-- Continued

- What's the difference between CML & SML?
- **CML**: *1.* Is an efficient set
  2. 'X' axis = σ; *3.* Only for efficient portfolios
- **SML:** *1.* Graphical representation of CAPM
  2. 'X' axis = β; *3.* For **all** securities and portfolios (efficient or inefficient)
- H.W. 1, 3, 6, 9, 11, 18, 21, 22(a,b), 25, 26, 30, 38

Review

- This chapter sets forth the principles of modern portfolio theory.
- The expected return and variance on a portfolio of two securities *A* and *B* are given by

$$E(r_P) = w_A E(r_A) + w_B E(r_B)$$

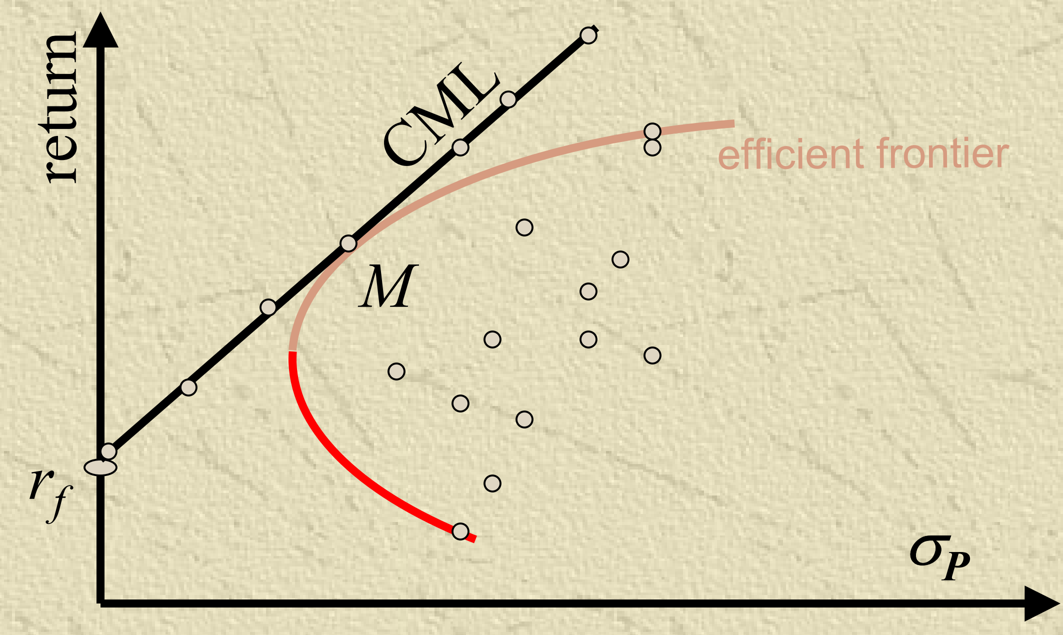
$$\sigma_P^2 = (w_A\sigma_A)^2 + (w_B\sigma_B)^2 + 2(w_B\sigma_B)(w_A\sigma_A)\rho_{AB}$$

  - By varying w_A, one can trace out the efficient set of portfolios. We graphed the efficient set for the two-asset case as a curve, pointing out that the degree of curvature reflects the diversification effect: the lower the correlation between the two securities, the greater the diversification.
  - The same general shape holds in a world of many assets.

Review-- Continued

- The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by the market portfolio.

- Then with borrowing or lending, the investor selects a point along the CML.



Review-- Concluded

- The contribution of a security to the risk of a well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution is called the beta.

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}$$

- The CAPM states that the expected return on a security is positively related to the security's beta:

$$\overline{R}_i = R_F + \beta_i \times (\overline{R}_M - R_F)$$