I. Give the implementation of DisjointSet. [30 pts]

```java
class DisjointSet
{

}
```

II. Give the truth table for the following propositions. [20 pts]

\[(p \rightarrow q) \rightarrow r\]
III. Use mathematical induction to prove \( n^2 > (2n + 1) \) for \( n \geq 4 \). [20 pts]

IV. Use Big-O, Big-Ω and Big-Θ notation to relate the following pairs: [5 pts each]

\[
\begin{array}{cc}
\text{n}^2 & \text{n}^2 + 2n \\
\log n & n/100 \\
2^n & 3^n \\
\text{n}^{2.5} & 2.5^n \\
\end{array}
\]

V. Give the composite of \( R_1 \) and \( R_2 \). To get partial credit, you like to show each step of your work. [20 pts]
\[
\begin{align*}
R_1 &= \{(a, b), (a, c), (b, c), (a, d)\} \\
R_2 &= \{(b, a), (c, b), (d, d)\}
\end{align*}
\]

VI. Convert the following proposition to CNF. [20 pts]
\!(p \rightarrow q) \lor (p \land q)

VII. Given a relation represented in a boolean matrix $M_{n \times n}$, implement a Java program to determine whether or not the relation is an equivalence relation. [30 pts]

VIII. Give the implementation of GCD(int m, int n). [20 pts]
IX. Give the implementation of BubbleSort(int[] a). [20 pts]