Consider the graph of a function \( y = f(x) \) and a point \((x_0, y_0)\) that is not on the graph. We wish to find the coordinates \((x, y)\) of the point(s) on the graph having the minimum distance the fixed point \((x_0, y_0)\). We also want to determine what this minimum distance is.

Recall that the distance formula between \((x, y)\) and \((x_0, y_0)\) is given by

\[
d = \sqrt{(x - x_0)^2 + (y - y_0)^2}.
\]

However the point \((x, y)\) is on the graph, so \(y = f(x)\). Thus the distance becomes solely a function of \(x\):

\[
d(x) = \sqrt{(x - x_0)^2 + (f(x) - y_0)^2}.
\]

We then can graph \(d(x)\) and find the point(s) at which the absolute minimum occurs using the built-in “minimum” command.

**Example 1.** Find the points on the parabola \(y = 4 - x^2\) that are at a minimum distance to the point \((0, -3)\). Then specify what that minimum distance is. Also illustrate the points on a graph of the parabola. Also show a separate graph of the distance function that is being minimized.

**Solution.** We first graph \(y = 4 - x^2\) and the fixed point \((0, -3)\). The vertex \((0, 4)\) on the graph is clearly 7 units away from the point \((0, -3)\). The graph then becomes closer to the point before eventually growing further away. By symmetry, there should be two points on the graph that are at a minimum distance.

The distance function to be minimized is

\[
d(x) = \sqrt{(x - 0)^2 + (4 - x^2 - (-3))^2} = \sqrt{x^2 + (7 - x^2)^2}, \text{ for all } x.
\]

We now plot this distance function in Y2. Observe that the distance to the point \((0, -3)\) is 7 when \(x = 0\). We can now use the “minimum” command.

The minimum distance occurs when \(x = \pm 2.54951\). The actual minimum distance is approximately 2.598.
Finally, we must find the $y$-coordinates of the points on the original graph that are at this minimum distance. After using the minimum command, the value of $X$ obtained is kept stored. Just enter the command $Y1$ on the Home screen to obtain the $y$-coordinate of $-2.5$.

Thus, the points $(\pm 2.54951, -2.5)$ are at a distance of about 2.598 from $(0, -3)$, and these are the closest points to $(0, -3)$ on the graph of $y = 4 - x^2$.

**Example 2.** Find the point on the graph of $y = \sqrt{x + 4}$ that is at a minimum distance to the point $(4, -2)$. Specify what that minimum distance is. Show the point on a graph of the function and show a separate graph of the distance function that is being minimized.

**Solution.** The distance function to be minimized is

$$d(x) = \sqrt{(x - 4)^2 + (\sqrt{x + 4} + 2)^2}, \text{ for } x \geq -4,$$

which is graphed in $Y2$ after $y = \sqrt{x + 4}$ is graphed in $Y1$. The minimum of the distance function occurs when $x \approx 3.1253761$ and the minimum distance to the point $(4, -2)$ is approximately 4.75. The point on the graph of $y = \sqrt{x + 4}$ where the min. distance occurs is approximately $(3.1253761, 2.66934)$. 

\[y = \sqrt{x + 4}\] and the point $(4, -2)$

Minimizing the dist. function

Evaluate $Y1$ to find point on graph

$y = \sqrt{x + 4}$