A small aircraft flies at a desired speed $V$ and a desired heading of $H$ degrees, where $H$ is measured to the North from the East (i.e., counterclockwise from the positive $x$-axis). The plane encounters a wind with speed $W$ heading to direction $D$ degrees (or coming from a particular direction).

**Result of Plane Encountering Wind**

If the plane maintains its speed and heading, what will be the resulting speed and heading upon encountering the wind?

Let $F_1$ be the force vector of the plane and $F_2$ be the force vector of the wind:

<table>
<thead>
<tr>
<th>$F_1 = (V \cos H, V \sin H)$</th>
<th>$F_2 = (W \cos D, W \sin D)$</th>
</tr>
</thead>
</table>

If the plane makes no adjustment, then the result is the vector $F_1 + F_2$.

**Adjustment to Keep Original Speed and Heading**

If the plane wants to maintain its original speed and heading, what new speed and heading should it take upon encountering the wind?

<table>
<thead>
<tr>
<th>$F_1 = (V \cos H, V \sin H)$</th>
<th>$F_2 = (W \cos D, W \sin D)$</th>
</tr>
</thead>
</table>

Now let $F_3$ be the adjusted course of the plane. With the wind, the result would be $F_3 + F_2$, which we want to equal the original desired course $F_1$. So let $F_3 + F_2 = F_1$ and therefore $F_3 = F_1 - F_2$ is the required adjusted course.

**Example 1.** (a) An plane flies in a direction $25^\circ$ South of West at 280 mph. It encounters a 30 mph wind that is heading $20^\circ$ East of North. Find the resulting speed and direction of the plane.

(b) If the plane encounters the wind described it Part (a), what speed and heading should it take in order to stay on its desired course of $25^\circ$ South of West at 280 mph?

**Solution.** (a) The plane’s heading is $205^\circ$ and the wind’s heading is $70^\circ$. The plane’s vector is

$$F_1 = (280 \cos 205^\circ, 280 \sin 205^\circ).$$

The wind’s vector is

$$F_2 = (30 \cos 70^\circ, 30 \sin 70^\circ).$$
Then \( F_1 + F_2 = (280 \cos205^\circ + 30 \cos70^\circ, 280 \sin205^\circ + 30 \sin70^\circ) \approx (-243.5, -90.14). \)

The resulting speed is about \( \sqrt{243.5^2 + 90.14^2} \approx 259.65 \) mph, and the resulting direction in Quad III is \( \theta = \tan^{-1}(90.14 / 243.5) + 180^\circ \approx 200.3^\circ \), or 20.3º South of West.

This headwind will slow the plane down from 280 mph to 259.65 mph, and will push the plane’s heading further North by almost 5º.

(b) Now let \( F_1 = (280 \cos205^\circ, 280 \sin205^\circ) \) be the desired course of the plane, and let \( F_2 = (30 \cos70^\circ, 30 \sin70^\circ) \) be the wind. Now \( F_3 \) is the new course that the plane will take to obtain \( F_1 \). Then \( F_3 + F_2 = F_1 \), so that \( F_3 = F_1 - F_2 \approx (-264.027, -146.524). \)

So the plane’s speed must be about \( \sqrt{264.027^2 + 146.524^2} \approx 301.96 \) mph, with a heading of \( \theta = \tan^{-1}(146.524 / 264.027) + 180^\circ \approx 209.028^\circ \), which is 29.028º South of West.

With this new speed and heading, the headwind it encounters will slow the plane down to 280 mph and push its course to 25º South of West.

**Example 2.** A plane flying 30º North of West at 300 mph meets a 25 mph tailwind coming from 10º South of East. What is the result?

\[
\begin{array}{c}
\text{Plane} \\
W \quad 30^\circ \\
N \quad 10^\circ \\
S \quad \text{Wind}
\end{array}
\]

*Solution.* The plane’s heading is 150º and its vector is \( F_1 = (300 \cos150^\circ, 300 \sin150^\circ) \).

If the wind is coming from 10º South of East, then it is heading to 10º North of West; that is, its direction is 170º. So the wind’s vector is \( F_2 = (25 \cos170^\circ, 25 \sin170^\circ) \).

The resulting vector is given by \( F_1 + F_2 = (-284.4278, 154.3412). \)

The resulting speed is \( \sqrt{284.4278^2 + 154.3412^2} \approx 323.6 \) mph, and the resulting angle is \( \theta = \tan^{-1}(-154.3412 / 284.4278) + 180^\circ \approx 151.514^\circ \), or just 1.514º off course.