The Equation of the Unit Circle:  \( x^2 + y^2 = 1 \)

Angle \( \theta \) is measured \textit{counterclockwise} from the positive \( x \)-axis. There are 360° in the entire circle. Angle \( \theta \) makes a point \((x, y)\) on the unit circle and an arclength of \( s \) units.

The \( x \)-coordinate is called the “cosine of \( \theta \)”:

The \( y \)-coordinate is called the “sine of \( \theta \)”:

\[
\begin{align*}
x &= \cos \theta \\
y &= \sin \theta 
\end{align*}
\]

The fundamental trigonometric identity is

\[\cos^2 \theta + \sin^2 \theta = 1\]

The \textit{radian measure} of angle \( \theta \) is the arclength \( s \) created by \( \theta \) on the unit circle.

The circumference of a circle is \( C = 2\pi r \). So the circumference of the unit circle is \( 2\pi \).

- Whole Circle \( \rightarrow \) 360° \( \rightarrow \) 2\(\pi\) radians
- Half-circle \( \rightarrow \) 180° \( \rightarrow \) \(\pi\) radians
- Quarter Circle \( \rightarrow \) 90° \( \rightarrow \) \(\frac{2\pi}{4} = \frac{\pi}{2}\) rad.
- Eighth of Circle \( \rightarrow \) 45° \( \rightarrow \) \(\frac{2\pi}{8} = \frac{\pi}{4}\) rad.
Converting Degrees to Radians

**One Way:** Take the whole circumference of $2\pi$, and multiply by the proportion of the circle being used: $2\pi \times \frac{z^\circ}{360^\circ}$

**Another Way:** Just multiply $z^\circ$ by $\frac{\pi}{180^\circ}$. (Because $2\pi \times \frac{z^\circ}{360^\circ} = z^\circ \times \frac{\pi}{180^\circ}$)

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**Example 1.** Convert $160^\circ$ to radians.

First way: $2\pi \times \frac{160^\circ}{360^\circ} = 2\pi \times \frac{16}{36} = \frac{32\pi}{36} = \frac{8\pi}{9}$ rad.

Second way: $160^\circ \times \frac{\pi}{180^\circ} = \frac{16\pi}{18} = \frac{8\pi}{9}$ rad.

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**Example 2.** Graph each angle below and show the orientation. Convert each angle to radians. For the negative angles, find positive angles (in both degrees and radians) that are co-terminal to the original negative angle.

(a) $270^\circ$          (b) $-135^\circ$    (c) $-220^\circ$

**Solution.** Negative angles are measured clockwise from the positive $x$-axis.

(a) Convert to radians: $270^\circ \times \frac{\pi}{180^\circ} = \frac{27\pi}{18} = \frac{3\pi}{2}$ radians

(b) $-135^\circ \times \frac{\pi}{180^\circ} = -\frac{3\pi}{4}$ rad. (after dividing out a common factor of 45)
To find a positive co-terminal angle (i.e., a positive angle that ends at the same point on the unit circle), simply add 360° or add $2\pi$:

Co-terminal positive angles: $-135° + 360° = 225°$ and $-\frac{3\pi}{4} + 2\pi = -\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{5\pi}{4}$

(c) Convert to radians: $-220° \times \frac{\pi}{180°} = -\frac{22\pi}{18} = -\frac{11\pi}{9}$ rad.

To find a positive co-terminal angle add 360° or add $2\pi = \frac{18\pi}{9}$:

Co-terminal positive angles: $-220° + 360° = 140°$ or $-\frac{11\pi}{9} + 2\pi = -\frac{11\pi}{9} + \frac{18\pi}{9} = \frac{7\pi}{9}$

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**Converting Radians to Degrees**

Given the radian measure of an angle, multiply it by $\frac{180°}{\pi}$ to convert back to degrees.

**Example 3.** Convert each angle to degrees. For the negative angles, find positive co-terminal angles (in both degrees and radians).

(a) $\frac{7\pi}{10}$  (b) $-\frac{11\pi}{6}$  (c) $-\frac{4\pi}{3}$

**Solution.** In each case, multiply by $\frac{180°}{\pi}$:

(a) $\frac{7\pi}{10} \times \frac{180°}{\pi} = 7 \times 18° = 126°$  (b) $-\frac{11\pi}{6} \times \frac{180°}{\pi} = -11 \times 30° = -330°$

(c) $-\frac{4\pi}{3} \times \frac{180°}{\pi} = -4 \times 60° = -240°$

Positive co-terminal angle for (b): $-330° + 360° = 30°$ or $-\frac{11\pi}{6} + \frac{12\pi}{6} = \frac{\pi}{6}$

(Add 360° or add $2\pi = \frac{12\pi}{6}$)

Positive co-terminal angle for (c): $-240° + 360° = 120°$ or $-\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{2\pi}{3}$

(Add 360° or add $2\pi = \frac{6\pi}{3}$)