Equation of the Circle: \( x^2 + y^2 = r^2 \)

If we have a circle of radius \( r \), then we multiply all variables by this factor of \( r \).

Now:

\[
x = r \cos \theta \quad \text{and} \quad y = r \sin \theta
\]

These formulas hold in any right triangle with hypotenuse \( r \), where \( x \) is “adjacent” to \( \theta \) and \( y \) is “opposite” of \( \theta \):
Arclength on the Circle of Radius $r$

Arclength $= s = \theta r$

where $\theta$ is in radians

**Note:** If $\theta$ is in degrees, then we first must convert to radians (by multiplying by $\frac{\pi}{180}$), then multiply by the radius $r$.

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**Example 1.** On a circle of radius 12 ft, find the arclength and the $(x, y)$ coordinates created by an angle of 60°.

**Solution.** First, convert 60° to radians $\rightarrow$

$60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3}$ (which would be the arclength on the *unit* circle). So on a circle of radius 12 ft, the arclength is

$$s = \theta r = \frac{\pi}{3} \times 12 \text{ ft} = 4\pi \text{ ft}$$

The $(x, y)$ coordinates are:

$$x = r \cos \theta = 12 \cos 60^\circ = 12 \times \frac{1}{2} = 6$$

$$y = r \sin \theta = 12 \sin 60^\circ = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

So $(x, y) = (6, 6\sqrt{3})$

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**Example 2.** On a circle of radius 5 inches, what angle (in degrees) in made by an arclength of 10 inches?

**Solution.** Again use $s = \theta r$, where $\theta$ is in radians. Then $10 \text{ in} = \theta \times (5 \text{ in})$, so the angle is $\theta = 2$ radians. To convert to degrees, multiply by $\frac{180^\circ}{\pi}$. Then

$$\theta = 2 \times \frac{180^\circ}{\pi} = \frac{360^\circ}{\pi} = 114.6^\circ$$