Let $A$ and $A'$ represent two sub-populations of a larger population $\Omega$. For example, let $\Omega = $ WKU students, which is broken down into $A = $ female and $A' = $ male.

Then let $B$ be a trait or characteristic. For example let $B = $ in-state student and $B' = $ out-of-state student.

The trait $B$ is independent of sub-population $A$ if $B$ has the same percentage among the whole population as it does among each sub-population $A$ and $A'$.

Assuming we know that $B$ is independent of $A$, then we can find the probability of any intersection by multiplying the proportions:

\[
P(A \cap B) = P(A) \times P(B) \quad P(A' \cap B) = P(A') \times P(B)
\]
\[
P(A \cap B') = P(A) \times P(B') \quad P(A' \cap B') = P(A') \times P(B')
\]

(We can only use these equations if we know that independence exists.)

**Example.** Suppose 60% of students are female, and 75% are in-state. Assume these traits are independent.

(a) Make a Block Diagram that shows all percentages.

(b) Among just females, what percentage are in-state?

(c) Among just males, what percentage are in-state?

(d) Among just females, what percentage are out-of-state?

(e) Among just males, what percentage are out-of-state?

(f) Among just in-state students, what percentage are female?

(g) Among just out-of-state students, what percentage are female?

(h) Among just in-state students, what percentage are male?

(i) Among just out-of-state students, what percentage are male?
Solution. Let $F = \text{Female}$, $F' = M = \text{Male}$, $I = \text{In-State}$, and $I' = O = \text{Out-of-State}$.

We know $P(F) = 0.60$ and $P(I) = 0.75$. By independence, we have

$$P(F \cap I) = P(F) \times P(I) = 0.60 \times 0.75 = 0.45$$

which is the additional value we need to complete the Blockdiagram.

<table>
<thead>
<tr>
<th></th>
<th>$I$</th>
<th>$O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>60%</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>75%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F$</th>
<th>$I$</th>
<th>$O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>45%</td>
<td>15%</td>
<td>60%</td>
</tr>
<tr>
<td>30%</td>
<td>10%</td>
<td>40%</td>
</tr>
<tr>
<td>75%</td>
<td>25%</td>
<td>100%</td>
</tr>
</tbody>
</table>

In-State: $P(I) = 75\%$  Out-of-State: $P(O) = 25\%$

Out of 100% total, 60% are female and 40% are male. So think of there being 60 females and 40 males.

(b) Among the 60 females, 45 are in-state; so $45/60 = 75\%$ of the females are in-state.

(c) Among 40 males, 30 are in-state; so $30/40 = 75\%$ of the males are in-state.

The percentages of in-state are the same regardless if you look at the whole population, look at just the females, or look at just the males. Thus, we see that being an in-state student is independent of whether or not you are male or female.

(d) Among 60 females, 15 are out-of-state; so $15/60 = 25\%$ of females are out-of-state.

(e) Among 40 males, 10 are out-of-state; so $10/40 = 25\%$ of males are out-of-state.

The percentages of out-of-state are the same regardless of which group we look at.

Now consider

<table>
<thead>
<tr>
<th></th>
<th>Female: $P(F) = 60%$</th>
<th>Male: $P(M) = 40%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Out of 100% total, 75% are in-state and 25% are out-of-state. So think of there being 75 in-state-students and 25 out-of-state students.

(f) Among 75 in-state students, 45 are female; so $45/75 = 60\%$ of in-state are female.

(g) Among 25 out-of-state, 15 are female; so $15/25 = 60\%$ of out-of-state are female.

The percentage of females is the same whether you look at the whole population, just look at in-state students, or just look at out-of-state students.

(h) Among 75 in-state students, 30 are male; so $30/75 = 40\%$ of in-state are male.

(i) Among 25 out-of-state, 10 are male; so $10/25 = 40\%$ of out-of-state are male.

The percentage of males is the same whether you look at the whole population, just look at in-state students, or just look at out-of-state students.
Exercise. Assume IQ levels are independent of height. Suppose 12% of the population has an IQ of at least 120, and 15% of the population is over 6 ft tall.

Let $I = \text{IQ} \geq 120$ and $T = \text{over 6 ft tall}$.

(a) Show the percentages on a Block diagram.

What percentage have

(b) IQ < 120 and over 6 ft tall?
(c) IQ $\geq 120$ or at most 6 ft tall?
(d) IQ < 120 or at most 6 ft tall, but not both?

(e) Among just those with IQ $\geq 120$, what percentage are over 6 ft tall?
(f) Among just those with IQ < 120, what percentage are over 6 ft tall?
(g) Among just those with IQ $\geq 120$, what percentage are at most 6 ft tall?
(h) Among just those with IQ < 120, what percentage are at most 6 ft tall?

(i) Among just those over 6 ft tall, what percentage have IQ $\geq 120$?
(j) Among just those at most 6 ft tall, what percentage have IQ $\geq 120$?
(k) Among just those over 6 ft tall, what percentage have IQ < 120?
(l) Among just those at most 6 ft tall, what percentage have IQ < 120?
Solution

\[ I = \text{IQ} > 120 \quad I' = \text{IQ} \leq 120 \quad T = \text{over 6 ft tall} \quad T' = \text{at most 6 ft tall} \]

\[ P(I \cap T) = 0.12 \times 0.15 = 0.018 \]

<table>
<thead>
<tr>
<th>( I )</th>
<th>( T )</th>
<th>( T' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8%</td>
<td>10.2%</td>
<td>12%</td>
</tr>
<tr>
<td>13.2%</td>
<td>74.8%</td>
<td>88%</td>
</tr>
<tr>
<td>15%</td>
<td>85%</td>
<td>100%</td>
</tr>
</tbody>
</table>

(b) IQ \leq 120 and over 6 ft tall: \( P(I' \cap T) \rightarrow 13.2\% \)

(c) IQ > 120 or at most 6 ft tall: \( P(I \cup T') \rightarrow 1.8\% + 10.2\% + 74.8\% \rightarrow 86.8\% \)

(d) IQ \leq 120 or at most 6 ft tall, but not both: \( P(I' \Delta T') = 13.2\% + 10.2\% = 23.4\% \)

(e) Among just those with IQ > 120, what pct are over 6 ft tall? By independence, still 15\%. (Or 1.8/12 \rightarrow 15\%)

(f) Among just those with IQ \leq 120, what pct are over 6 ft tall? By independence, still 15\%. (Or 13.2/88 \rightarrow 15\%)

(g) Among just those with IQ > 120, what pct are at most 6 ft tall? By independence, still 85\%. (Or 10.2/12 \rightarrow 85\%)

(h)Among just those with IQ \leq 120, what pct are at most 6 ft tall? By independence, still 85\%. (Or 74.8/88 \rightarrow 85\%)

(i) Among just those over 6 ft tall, what pct have IQ > 120? By independence, still 12\%. (Or 1.8/15 \rightarrow 12\%)

(j) Among just those at most 6 ft tall, what pct have IQ > 120? By independence, still 12\%. (Or 10.2/85 \rightarrow 12\%)

(k) Among just those over 6 ft tall, what pct have IQ \leq 120? By independence, still 88\%. (Or 13.2/15 \rightarrow 88\%)

(l) Among just those at most 6 ft tall, what pct have IQ \leq 120? By independence, still 88\%. (Or 74.8/85 \rightarrow 88\%)