The hands in poker which have value are a Pair, Two Pair, Three of a Kind, Straight, Full House, Flush, Straight Flush, Four of a Kind, and Royal Flush. We will describe these hands and count how many ways they can occur in a standard five-card deal.

**Values and Suits**

There are 13 possible values: The nine number cards 2 – 10, the three face cards J, Q, and K, and the Ace. Each value occurs in 4 suits: Spades, Hearts, Diamonds, and Clubs. So there are $13 \times 4 = 52$ cards.

To deal a hand, we simply choose 5 of the cards. Thus, there are $C(52, 5) = 2,598,960$ possible poker hands.

**Royal Flush**

```
          10
       ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
  ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣

      J
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣

      Q
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣

      K
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣

      A
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣
   ♣ ♣ ♣ ♣ ♣
     ♣ ♣ ♣ ♣ ♣
```

A royal flush contains the values 10, J, Q, K, A all in the same suit. Above is the royal flush in Clubs. Because there are 4 suits, there are 4 royal flushes.

**Straight Flush**

```
      7
  ♥ ♥ ♥ ♥ ♥
   ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥

      8
  ♥ ♥ ♥ ♥ ♥
   ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥

      9
  ♥ ♥ ♥ ♥ ♥
   ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥

      10
  ♥ ♥ ♥ ♥ ♥
   ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥

      J
  ♥ ♥ ♥ ♥ ♥
   ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
  ♥ ♥ ♥ ♥ ♥
```

A straight flush is a hand having 5 cards all in the same suit and in consecutive sequence (excluding the royal flushes, which are already counted). Above is a Jack-high straight flush in Hearts.

The Ace can count as a low card for a straight flush. Thus, the straight flushes can be A – 5, 2 – 6, . . . , or 9 – K in any of the four suits. There are four possible suits and nine possibilities for the first value (then the other four values are determined automatically). So there are $4 \times 9 = 36$ straight flushes.
A flush is a hand having 5 cards all in the same suit (excluding the straight flushes and the royal flushes, which are already counted). Above is a flush in diamonds. To count the total number of arbitrary flushes, we pick a suit, then pick 5 values. Then we exclude the 36 straight flushes and the 4 royal flushes. Therefore, there are actually \( C(4,1) \times C(13,5) - 40 = 4 \times 1287 - 40 = 5,108 \) flushes.

A straight is a hand having 5 cards in sequence (excluding the straight flushes and royal flushes, which are already counted). Above is a Queen-high straight. The Ace can count as a low card or a high card. Thus, the straight can be A – 5, 2 – 6, . . . , 9 – K, or 10 – A, but not all in the same suit.

To count the total number of arbitrary straights, we pick a first value from the 10 possible first values (the remaining four values are determined automatically), then we pick a suit for each value. Then we exclude the 36 straight flushes and the 4 royal flushes. So there are actually 10 \times 4 \times 4 \times 4 \times 4 \times 4 - 40 = 10,200 \) straights.
Pair, Two Pair, Three of a Kind, Full House, Four of a Kind

Step 1: From the 13 values, choose the number of values needed.
Step 2: From the number of values chosen in Step 1, choose those needed to create the hand (i.e., to be doubled, tripled, etc.)
Step 3: Choose the required suits for the values chosen in Step 1.

**Pair**

Choose Values × Choose Pair × Suit × Suits × Suit × Suit

**Two Pair**

Choose Values × Choose Two Pair × Suits × Suits × Suit
### Three of a Kind

<table>
<thead>
<tr>
<th>4</th>
<th>8</th>
<th>8</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>♥</td>
<td>♣</td>
<td>♦</td>
<td>♠</td>
<td>♦</td>
</tr>
<tr>
<td>♥</td>
<td>♣</td>
<td>♦</td>
<td>♠</td>
<td>♦</td>
</tr>
</tbody>
</table>

Choose Values × Choose Triple × Suit × Suits × Suit

### Full House

<table>
<thead>
<tr>
<th>9</th>
<th>9</th>
<th>J</th>
<th>J</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>♠</td>
<td>♦</td>
<td>♥</td>
<td>♣</td>
<td>♦</td>
</tr>
<tr>
<td>♠</td>
<td>♦</td>
<td>♥</td>
<td>♣</td>
<td>♦</td>
</tr>
</tbody>
</table>

Choose Values × Choose Triple × Suits × Suits

### Four of a Kind

<table>
<thead>
<tr>
<th>9</th>
<th>Q</th>
<th>Q</th>
<th>Q</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>♠</td>
<td>♠</td>
<td>♥</td>
<td>♠</td>
<td>♠</td>
</tr>
<tr>
<td>♠</td>
<td>♠</td>
<td>♥</td>
<td>♠</td>
<td>♠</td>
</tr>
</tbody>
</table>

Choose Values × Choose Quadruple × Suit × Suits
<table>
<thead>
<tr>
<th>Hand</th>
<th>Number</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal Flush</td>
<td>4</td>
<td>0.000001539</td>
</tr>
<tr>
<td>Nothing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Hands</td>
<td>$C(52, 5) = 2,598,960$</td>
<td>1</td>
</tr>
</tbody>
</table>