Are Changes in the Monetary Base More Effective When They Result from a Change in the Central Bank’s Nominal Target?

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Abstract

Over the last few years, the Federal Reserve has increased the monetary base fourfold. Given the Federal Reserve’s dual mandate, the objective of this policy has been to generate an increase in real economic activity while maintaining a low, stable rate of inflation. Some, such as Sumner (2011, 2012) and Woodford (2012) have advocated making the expansion of the monetary base conditional upon a particular target for nominal income. This paper develops a model to examine the effects of exogenous changes in the monetary base and changes to the monetary base that result from exogenous changes to the central bank’s nominal target. The model is estimated using Bayesian estimation. The posterior distribution of impulse response functions suggests that changes in the monetary base that result from changes in the nominal target are larger and more persistent that exogenous changes in the monetary base itself.

Keywords monetary policy, nominal income expectations, quantitative easing

JEL Classification E52, E58

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1 Introduction

Over the last few years, the Federal Reserve has increased the monetary base fourfold. Given the Federal Reserve’s dual mandate, the objective of this policy has been to generate an increase in real economic activity while maintaining a low, stable rate of inflation. A number of observers, such as Romer (2011), Hatzius et al. (2011), Evans (2011), and Williams (2014) have questioned the effectiveness of this approach and have suggested that the Federal Reserve target nominal income. Some, such as Sumner (2011, 2012) and Woodford (2012) have advocated making the expansion of the monetary base conditional upon a particular target for nominal income. For example, Woodford (2012: 48) argues that “without clarifying the target criterion that is assumed to shape future policy deliberations – or referring only to purely forward-looking criteria, that do not incorporate the kind of commitment to correction of target misses that a nominal GDP target path would imply – accomplishes very little.” The Sumner-Woodford proposals raise several questions. First, what is the mechanism under which such a policy could be successful? Second, would a change in the target for nominal income result in the desired outcome of an increase in real economic activity? Third, would a change in the monetary base that results from a change in the central bank’s target have a larger and more significant effect on economic activity than an exogenous change in the monetary base?

The purpose of this paper is to address these questions. The monetary transmission mechanism question in particular is key to understand and evaluate the Sumner-Woodford proposal. In this paper we propose a monetary transmission mechanism in which the effects of monetary policy are transmitted through changes in the monetary base. While the changes in the monetary base have a direct effect on economic activity, the changes also produce a secondary effect. Specifically, it is assumed that financial intermediary is a function of nominal income. As a result, the increase in nominal income that results from the change in the monetary base produces a secondary effect on economic activity as a result of the change in intermediation.

Monetary policy shocks, however, can occur for two different reasons. First, it is possible for the monetary policy shock to be characterized by a shock to the monetary base. In this case, the shock to

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Woodford differs from Sumner in the following respect. Sumner (2012) advocates using the monetary base to hit the central bank’s nominal income target. Woodford (2012) argues that a successful policy is one that establishes an explicit target for providing forward guidance about the central bank’s interest rate policy coupled with actions taken to immediately stimulate spending. He stresses, however, that successful actions are likely to be targeted purchases of particular assets rather than traditional increases in the supply of bank reserves. Nonetheless, their case for attaching policy actions to a nominal income target is similar.
the monetary base represents an unanticipated deviation from the monetary policy rule. Second, it is possible that the monetary base can change as the result of a change in the target of monetary policy. The model used this paper assumes that monetary policy is subject to both types of shock. Thus, the model is able to distinguish between monetary policy shocks that represent deviations from the policy rule and monetary policy shocks that result from changes in the central bank’s target.

The model used in this paper is an extension of the monetary search framework of Lagos and Wright (2005). The monetary search framework is modified in three important ways. For example, the types of financial assets are expand to include not only fiat currency, but also deposits. In addition, whereas economic agents move sequentially through different markets in the standard monetary search framework, the present model follows Williamson (2006) in assuming that economic agents enter one of two markets each period, that these markets operate simultaneously, and that agents move probabilistically across markets over time. Monetary injections take place in one market. This implies that only a subset of economic agents receive the monetary injection. As a result, this increases demand in this market, but the price level will not fully adjust to the monetary injection even though prices are flexible. Monetary policy therefore has an effect on real economic activity. This setup therefore draws on the institutional fact that monetary policy is conducted through financial markets via open market operations. In this sense, the setup bears resemblance to the limited participation framework of Lucas (1990) and Fuerst (1992). In contrast with those frameworks, however, the effects of monetary policy in the present context are persistent.

The framework is also extended to include financial intermediaries. The motivation for intermediation is derived from the fact that some agents experience idiosyncratic productivity shocks. Economic agents that wish to smooth consumption will therefore prefer to borrow before the productivity shock is realized. As outlined below, this assumption implies that agents that experience an adverse productivity shock might have an incentive to default on their loan. Banks therefore impose a borrowing constraint that is a function of the expected nominal income of the borrower. In the event of default, the bank pays a monitoring cost to seize the output of the borrower. If coupled with the limited participation assumption outlined above, it can be shown that the collateral constraint is always binding. This is important because when this constraint is binding, expected changes in nominal income affect the creation of bank deposits and economic activity.

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2This assumption is similar to that of Carlstrom and Fuerst (1997).
Finally, it is assumed that monetary policy is conducted by a central bank that adjusts the money supply to deviations of nominal GDP from a particular target. In addition, the target for nominal GDP is considered exogenous. This particular feature of the model is important because an increase in the central bank’s nominal GDP target requires an increase in the monetary base according to the monetary rule. Thus, it is possible to consider the effects of two different shocks to the monetary policy rule.

In order to empirically evaluate the effects of the monetary policy shocks, the model is estimated using Bayesian estimation. Direct estimation of the model enables one to consider the relative size of the two shocks. The posterior distribution of the impulse response functions of the two different monetary policy shocks are then used to evaluate the relative effects of each shock. The results show that the standard deviation of shocks to the nominal GDP target are over ten times as large as the shocks to the monetary base. The impulse response functions show that positive shocks to both the monetary base and to the nominal GDP target result in increases in nominal GDP, a broad measure of money, and real GDP. However, the results are larger and more persistent for the shock to the nominal GDP target than for the shock to the monetary base. Overall, these results provide support for the claims made by Sumner (2011, 2012) and Woodford (2012) that policy would be more effective if changes in the monetary base were the result of a change in the central bank’s target.

2 The Model

Time is discrete and continues forever. There is a continuum of infinitely-lived agents with unit mass. Each period consists of two subperiods. In the first subperiod, agents can make a deposit with or take out a loan from a financial intermediary. In second subperiod, agents interact in one of two markets that operate simultaneously. The first market is a centralized, or Walrasian market, henceforth referred to as the CM. The second market is a decentralized market (DM) in which agents are matched pairwise to trade. Each period a fraction of agents, $\pi/(1+\pi)$, are in the decentralized market. The remaining agents are in the centralized market. Agents in the CM enter the DM in the next period with probability $\pi$ such that the fraction of agents in the DM is constant across time. Upon entering the DM, agents receive an idiosyncratic preference shock. Specifically, with probability $\sigma$ agents are buyers who are matched pairwise with a seller. Symmetrically, with probability $\sigma$ agents are sellers who are matched pairwise.

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3Hendrickson (2012) suggests that the Federal Reserve was effectively targeting nominal income during the Great Moderation.
with a buyer. Finally, with probability \(1 - 2\sigma\), agents are not matched. Once matched, buyers and sellers negotiate the terms of trade. In the DM, agents do not have access to the trading histories of previous agents and there is no record-keeping technology. As such, a medium of exchange is essential.\(^4\) There are two assets that can serve as medium of exchange. The first is a fiat currency, which is intrinsically useless. Claims to deposits with the financial intermediary represent the second asset capable of serving as medium of exchange. More will be said on deposit claims below.

Agents can produce and consume in the CM. The production technology for the CM good is linear, \(y_t(i) = z_t(i)h_t(i)\), where \(y\) denotes the output of the CM good, \(h\) denotes labor, and \(z(i)\) is an idiosyncratic productivity shock for agent \(i \in \{l, h\}\), where \(l\) denotes a low productivity type and \(h\) represents a high productivity type. An agent’s ability to consume in the CM is dependent on income from production and the level of asset balances. As a result of the idiosyncratic productivity shock, agents entering the CM in each period have an incentive to borrow from the financial intermediary to smooth consumption. However, agents that experience an adverse productivity shock have an incentive to default on their loan. The financial intermediary responds to this incentive by imposing a borrowing constraint and monitoring the borrower. If the borrower defaults, the intermediary seizes the output of the borrower and transfers it to depositors.

Agents entering the DM do not have an incentive to borrow from the financial intermediary for two reasons. First, agents do not know in advance whether they are buyers or sellers. Second, even if agents knew their “type” before entering the DM, sellers would not have an incentive to borrow because they produce and do not consume. Symmetrically, buyers do not produce in the DM. As a result, a loan would actually reduce the set of feasible allocations relative to using money and/or deposits given the interest payment associated with the loan. Agents entering the DM, however, do have an incentive to deposit currency with the financial intermediary since, in the event that the agent is unmatched in the DM, deposits earn interest and currency does not. In addition, as noted above, deposits circulate as a medium of exchange in the DM. However, it is assumed that claims to deposits are imperfectly recognizable and therefore there is a threat of fraud.\(^5\) It can be shown that while no fraud will actually occur in equilibrium, the threat of fraud is sufficient to explain why agents are indifferent between holding

\(^4\)A medium of exchange is essential in the sense described in Kocherlakota (1998) in that a medium of exchange enlarges the set of feasible allocations.

\(^5\)Some have questioned whether it is reasonable to assume that issues of fraud and counterfeiting are applicable to deposits. According to the Philadelphia Federal Reserve, the combined losses to consumers, merchants, and the financial industry due to check fraud is between $5 - 10 billion per year (Federal Reserve Bank of Philadelphia Annual Report, 2005: 20 - 21).
currency and deposits.

Formally, the preferences of agents in the model are given as

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u(q_t) - c(q_t) + x_t - v(h_t)] \]

where \( u(q_t) \) denotes the utility associated with consuming a quantity, \( q_t \), of the good in the DM, \( c(q_t) \) is the cost of producing a quantity \( q_t \) of the DM good, \( x_t \) is the quantity of the CM good consumed and \( v(h_t) \) measures the disutility of labor, \( h_t \). In addition, it is assumed that \( u(0) = 0, u', -u'', c' > 0, c'' \geq 0, u'(0) = \infty, \) and \( u'(\infty) = 0. \)

Finally, there is a central bank that controls the supply of currency. The central bank is assumed to control the supply of currency through lump sum transfers to agents in the centralized market. Specifically, it is assumed that the central bank adjusts the supply of currency to fluctuations in nominal NGDP relative to some target.

The analysis begins in the second subperiod and works backward.

### 2.1 The Centralized Market

Let \( \phi_t \) denote the price of money in terms of the CM good. The budget constraint for agents entering the CM is given as

\[ x_t = \phi_t(m_t + t_t) + (1 + i_{d,t})\phi_t d_t - i_{d,t}\phi_t \ell_t - \phi_t m_{t+1} + z_t(i) h_t(i) \]  

where \( m_t \) is money balances, \( t_t \) is the lump sum transfer from the central bank, \( h_t \) is the quantity of labor, \( \ell_t \) is the quantity borrowed from the financial intermediary, \( d_t \) are the deposits made with the financial intermediary, and \( z_t \in \{z_l, z_h\} \) is an idiosyncratic measure of productivity. Specifically, \( z_t \) is given as

\[ z_t = \begin{cases}  
  z_h & \text{with probability } \ p \\
  z_l & \text{with probability } \ 1 - p 
\end{cases} \]

The value function for those entering the centralized market satisfies

\[ W_t(m_t, d_t) = \max_{x_t, h_t, \ell_t, d_{t+1}, m_{t+1}} \left[ x_t - v(h_t) + \pi \beta V_{t+1}(m_{t+1}, d_{t+1}) + (1 - \pi) \beta W_{t+1}(m_{t+1}, d_{t+1}) \right] \]
Assuming an interior solution, one can solve equation (1) for $x_t$ and substitute this into the CM value function such that

$$W_t(m_t, d_t) = \max_{h_t, \ell_t, d_{t+1}, m_{t+1}} \left[ \phi_t(m_t + t_t) + (1 + i_{d,t}) \phi_t d_t - i_{\ell,t} \phi_t \ell_t + z_t(i) h_t - \phi_t m_{t+1} 
$$

$$- v(h_t) + \pi \beta W_{t+1}(m_{t+1}, d_{t+1}) + (1 - \pi) \beta W_{t+1}(m_{t+1}, d_{t+1}) \right]$$

(2)

In addition, since agents in the CM have an incentive to default in the event of an adverse productivity shock, the intermediary imposes a constraint such that the value of the loan cannot be greater than the nominal value of the agent’s expected income. Formally, this constraint is given as:

$$E_{t-1}(y_t/\phi_t) \geq \ell_t$$

(3)

### 2.2 The Decentralized Market

When agents enter the DM they receive a preference shock such that with probability $\sigma$ the agent is a buyer that is matched with a seller, with probability $\sigma$ the agent is a seller matched with a buyer, and with probability $1 - 2\sigma$ the agent is unmatched. The value function for agents entering the DM is therefore given by

$$V_t(m_t, d_t) = \sigma [u(q_t) + \beta E_t W_{t+1}(m_t - a_{m,t}, d_t - a_{d,t})] + \sigma [-c(q_t) + \beta E_t W_{t+1}(m_t + a_{m,t}, d_t + a_{d,t})]$$

$$+ (1 - 2\sigma) \beta E_t W_{t+1}(m_t, d_t)$$

(4)

where $0 < a_{m,t} \leq m_t$ and $0 < a_{d,t} \leq d_t$ denote the quantities of money and deposits offered in exchange for the DM good, respectively.

Once buyers and sellers are matched, they negotiate the terms of trade. Since trade is anonymous in the DM, buyers offer a medium of exchange, or some combination of media of exchange, for the DM good. For simplicity, it is assumed that buyers make take-it-or-leave-it offers to sellers. This offer must satisfy:

$$\phi_t(a_{m,t} + a_{d,t}) \geq c(q_t)$$

Since the buyer makes a take-it-or-leave-it offer and has an incentive to maximize the surplus, this
condition will always be binding in equilibrium. It is assumed below that the supply of currency grows at a rate higher than minus the rate of time preference. This implies that money is costly to hold and therefore $a_{m,t} = m_t$. Agents can offer claims to deposits as well. However, deposit claims are not perfectly recognizable and therefore there is a threat of counterfeiting. The cost of counterfeiting is assumed to be fixed and equal to $\kappa$ and is known by all agents. It follows that no agent will accept a deposit claim unless the value of the claim is less than or equal to the cost of counterfeiting. Thus, the take-it-or-leave-it offer satisfies

$$\phi_t(m_t + a_{d,t}) \geq c(q_t)$$

$$\kappa \geq \phi_t a_{d,t}$$

$$0 < a_{d,t} \leq d_t$$

Given that $\kappa$ is assumed to be known by all agents, (6) ensures that no counterfeiting takes place in equilibrium.

Using the linearity of $W$, (4) can be re-written

$$V_t(m_t, d_t) = \sigma[u(q_t) - c(q_t)] + \beta E_t W_{t+1}(m_t, d_t)$$

Iterating the DM value function forward and substituting into (2), the all agents seek to maximize equation (2) subject to (6), (7), and (3) at the beginning of each period. Formally, the maximization problem can be written

$$\max_{h_t, \ell_{t+1}, m_{t+1}, d_{t+1}, a_{d,t+1}} \left\{ \phi_t(m_t + t_l) + (1 + i_{d,t})d_t - i_{\ell,t} \phi_t \ell_t + z_t(i)h_t(i) - \phi_t m_{t+1} - v[h_t(i)] \right\}$$

$$+ \pi [\sigma[u(q_{t+1}) - c(q_{t+1})] + \beta E_t W_{t+2}(m_{t+1}, d_{t+1})] + (1 - \pi) \beta E_t W_{t+1}(m_{t+1}, d_{t+1})$$

$$+ \lambda_t \left( \frac{\kappa}{\phi_{t+1}} - a_{d,t+1} \right) + \Lambda_t (d_{t+1} - a_{d,t+1}) + \Theta_t (E_{t-1}y_t/\phi_t - \ell_t)$$

where $\lambda_t, \Lambda_t, \Theta$ are Lagrangian multipliers.

Assuming for simplicity that both assets are held in equilibrium and that the constraints are binding,
the equilibrium conditions are given as

\[ z_t(i) = u'[h_t(i)] \tag{8} \]

\[ \phi_t = \sigma \pi \beta E_t \phi_{t+1} \left[ \frac{u'(q_{t+1})}{c'(q_{t+1})} - 1 \right] + \pi \beta^2 E_t \phi_{t+2} + (1 - \pi) \beta E_t \phi_{t+1} \tag{9} \]

\[ \Theta_t = i_{t,t} \phi_t \tag{10} \]

\[ \pi \beta^2 E_t (1 + i_{d,t+2}) \phi_{t+2} + (1 - \pi) \beta E_t (1 + i_{d,t+1}) \phi_{t+1} + \Lambda_t = 0 \tag{11} \]

\[ \pi \sigma \beta E_t \phi_{t+1} \left[ \frac{u'(q_{t+1})}{c'(q_{t+1})} - 1 \right] = \lambda_t + \Lambda_t \tag{12} \]

\[ [p z_t h_t + (1 - p) z_t h_t] E_{t-1} (1/ \phi_t) = \ell_t \tag{13} \]

\[ a_{d,t} = d_t \tag{14} \]

\[ \kappa = \phi_t a_{d,t} \tag{15} \]

2.3 The Financial Intermediary

As noted above, borrowers in the CM potentially have an incentive to default if they experience an adverse productivity shock. If an agent defaults, the financial intermediary pays a monitoring cost proportional to the size of the loan, \( \theta \ell_t \), and seizes the output of the agent and transfers them to depositors. Thus, an agent experiencing an adverse productivity shock will have an incentive to default if

\[ z_t h_t \leq \ell_t \]

Assuming that (3) is binding, an agent will default any time they experience an adverse productivity shock.

The financial intermediary’s objective is to maximize profit, which is given as

\[ \Pi = p \phi_t (1 + i_{\ell,t}) L_t + (1 - p) \phi_t (z_t h_t - \theta) - \phi_t (1 + i_{d,t}) D_t \]

where \( \Pi \) denotes profit, \( L_t \) is the total quantity of loans, \( D_t \) is the total quantity of deposits, \( \theta \) is the monitoring cost, which is assumed to be constant over time, and \( i_{\ell,t} \) and \( i_{d,t} \) are the interest rates on loans and deposits, respectively. Given that the intermediary is only operable in the first subperiod, it is
not possible for agents to make a withdraw until the subsequent period. As such, the intermediary will
not hold reserves, which implies that $D_t = L_t$. The intermediary chooses $L_t$ to maximize profit. The
profit maximizing condition for the intermediary is given as:

$$p(1 + i_{\ell,t}) = (1 + i_{d,t})$$  \hspace{1cm} (16)$$

The spread between interest rates is therefore a function of the probability of default.

2.4 Closing the Model

2.4.1 Market-Clearing Conditions

The equilibrium in the market for the CM good is given as

$$[p z_h + (1 - p) z_l] H_t = x_t = X_t$$  \hspace{1cm} (17)$$

where $H = p h(h) + (1 - p) h(l)$ and $X_t$ are the aggregate quantity of labor, and the aggregate quantity
of the CM good, respectively.

Given the condition above and the fact that $M_t = M_{t-1} + T_t$, the aggregate resource constraint in
the CM is given as

$$\left(\frac{1}{1 + \pi}\right) (1 + i_{d,t}) \phi_t D_t = i_{\ell,t} \phi_t L_t$$  \hspace{1cm} (18)$$

Aggregating (13) yields

$$[p z_h + (1 - p) z_l] H_t E_{t-1}(1/\phi_t) = L_t$$  \hspace{1cm} (19)$$

From (5) and (14),

$$\left(\frac{\pi}{1 + \pi}\right) \phi_t [M_t + D_t] = c(q_t)$$  \hspace{1cm} (20)$$

From the financial intermediary, it is true in equilibrium that

$$D_t = L_t$$  \hspace{1cm} (21)$$
2.4.2 Accounting

Given the definitions of the model, nominal GDP is defined as

\[ NGDP_t = \left( \frac{\pi}{1 + \pi} \right) (M_t + D_t) + X_t/\phi_t \]  

(22)

It follows that real GDP is given as

\[ RGDP_t = \phi_t NGDP_t \]  

(23)

Finally, the broad measure of the money supply, \( M_t^A \) can be defined as

\[ M_t^A = M_t + D_t \]  

(24)

2.4.3 The Central Bank

The central bank is assumed to have the following rule

\[ \ln(M_t) = \rho_m \ln(M_{t-1}) - \alpha (1 - \rho_m) \ln(NGDP_t/NGDPT_t) + \varepsilon_t^M \]  

(25)

where \( \rho_m \in (0, 1) \) and \( \alpha \) are parameters, \( NGDPT \) is the central bank’s target for nominal GDP, and \( \varepsilon^M \) is a shock to the monetary base.

In addition, it is assumed that the central bank’s nominal GDP target follows the process

\[ \ln(NGDPT_t) = \rho_n \ln(NGDPT_{t-1}) + \varepsilon_t^N \]  

(26)

where \( \rho_n \in (0, 1) \) is a parameter and \( \varepsilon^N \) is a shock to the central bank’s nominal GDP target.

2.4.4 Equilibrium

Given \( p, z_t, z_h \), equations (8) - (12) and (16) - (26) are sufficient to solve for \( H_t, X_t, \phi_t, q_t, \Theta_t, i_{t,t}, i_{d,t}, \Lambda_t, \lambda_t, L_t, D_t, M_t, M_t^A, NGDP_t, RGDP_t, \) and \( NGDPT_t \).
3 Empirical Approach

In the model above, there are two possible sources of exogenous fluctuations in the monetary base. The first is a deviation of the monetary base from the level implied by the central bank’s nominal GDP targeting rule. Second is a change in the monetary base that results from a change in the nominal GDP target. Specifically, if the nominal GDP target increases, this causes the central bank to increase the monetary base in proportion to the change in the target. In this section, the model is estimated using Bayesian estimation and the posterior distribution of impulse response functions to both shocks are used to evaluate both types of fluctuations in the monetary base.

3.1 Functional Forms

Within the model, there are three functions that need to be assigned specific functional forms in order to estimate the model. It is assumed that the utility of DM consumption is given as \( u(q) = q^{1-\gamma}/(1 - \gamma) \). The disutility of production is given as \( c(q) = q \). Finally, the disutility of labor in the CM is given as \( v[h(i)] = h(i)^{1+\epsilon}/(1 + \epsilon) \). This implies that \( h(h) = z_h^{1/\epsilon} \) and \( h(l) = z_l^{1/\epsilon} \) and therefore \( H_t = pH(h) + (1 - p)h(l) \). Given that \( p, z_h, \) and \( z_l \) are assumed to be constant, the supply of labor is constant as well. Given that market-clearing implies that \( [pz_h + (1 - p)z_l]H = X, X \) is constant as well. For simplicity, \( X \) is normalized to one.

3.2 Estimation Details

The model is estimated using Bayesian estimation.\(^6\) Specifically, consider that the model presented above has a state space representation

\[
Y_t = \Gamma S_t
\]

\[
S_t = \Phi S_{t-1} + \Psi \nu_t
\]

where \( \Gamma, \Phi, \) and \( \Psi \) are matrices that are functions of the structural parameters of the model, \( Y_t \) are the control variables, \( S_t \) are the state variables, and \( \nu \) are the structural shocks of the equilibrium system.

Define a new vector of state variables as \( S_t = [Y_t', S_t'] \) and the vector of observable variables as \( Y_t \). Using this definition of the state variables and the observable variables, the state space representation of

\(^6\)For an overview of Bayesian estimation, see An and Schorfheide (2007).
the model presented above is given as

\[ S_t = AS_{t-1} + B\nu_t \]

\[ Y_t = C + DS_t + \xi_t \]

where \( A \) and \( B \) are function of \( \Gamma \), \( \Phi \), and \( \Psi \) defined above, \( C \) is a vector to match the means of the observables, \( D \) is a matrix of zeros and ones used to match the observables with the model definitions, and \( \xi \) is a vector of measurement errors. Let \( \Xi \) denote the vector of parameters in the model. The likelihood function of the model is therefore given as

\[ \mathcal{L}(Y_T|\Xi) = \prod_{t=1}^{T} \mathcal{L}(Y_t|Y_{1,...,t-1},\Xi) \]

The state space representation outlined above means that the Kalman filter can be used to compute the likelihood function of the model.

The posterior distribution of the model parameters is characterized using the Metropolis-Hastings algorithm, which can be described as follows. The Metropolis-Hastings algorithm starts with an initial parameter vector \( \Xi_{1,0} \), which is used by the Kalman filter to obtain an estimate of \( \mathcal{L}(Y_T|\Xi_{1,0}) \). This initial parameter vector is then updated using the Metropolis-Hastings random walk law of motion

\[ \Xi_{1,1} = \Xi_{1,0} + \omega \psi \epsilon_1 \]

where \( \Xi_{1,1} \) is the updated parameter vector, \( \omega \) is the “jump” scalar, \( \psi \) is the choleski decomposition of the variance-covariance matrix of \( \Xi_{1,1} \), and \( \epsilon_1 \) is distributed normal.

The updated parameter vector is then inputed into the Kalman filter to calculate the likelihood function of the model. The Metropolis-Hastings decision rule determines whether to accept the initial parameter vector or the proposed update. This process is carried out \( N \) times and used to generate a posterior density, \( p(\Xi|Y_T) \). The importance of the law of motion is paramount to estimation. In particular, the parameter \( \omega \) determines the size the of jump in the Metropolis-Hastings random walk update. The larger the size of the jump parameter, the lower the acceptance rate in the algorithm. In this paper, we choose the jump parameter such that the acceptance ratio is 25%. This acceptance ratio
ensures that the iterations both do not end up in the tails of the distribution and that the iterations do not end up in a subspace of the distribution.

All of the parameters of the model are estimated with the exception of the search friction parameter, \( \sigma \), which is set to 0.5 and the probability of a high productivity shock, \( p \), which is set to 0.75. This value for \( \sigma \) implies that every individual in the decentralized market will trade. The prior beliefs about the distribution of the parameters are listed in Table 1 and plotted in Figure 1. The prior mean of the discount factor, \( \beta \), is assumed to follow a Beta distribution with mean 0.99 and standard deviation of 0.001. This reflects the views that the discount factor should be between zero and one and that the prior belief about \( \beta \) is relatively tight around this prior. The probability of agents in the CM entering the DM next period, \( \pi \), is assumed to follow a Beta distribution with mean 0.50 and standard deviation 0.25. This is a relatively flat prior that reflects the fact that this is a relatively stylized parameter in the model. The parameter \( \gamma \) is assumed to have a Gamma distribution with mean 2.00 and standard deviation 0.25. The responsiveness of the central bank to deviations of nominal GDP from trend, \( \alpha \), is assumed to follow a Gamma distribution with a mean of 1.50 and a standard deviation 0.25. This prior reflects the view that the central bank “leans against the wind” by reducing the monetary base by more than the deviation of nominal GDP from trend, but that it is unlikely that this parameter is larger than 2.00. The autoregressive parameters for the monetary base, \( \rho_m \), and the nominal GDP target, \( \rho_n \), are assumed to follow a Beta distribution with mean 0.50 and standard deviation of 0.25. This assumption represents a relatively flat prior for the autoregressive parameters. Finally, the model is log-linearized around the steady state and therefore \( \sigma_m \) and \( \sigma_n \) are assumed to have an Inverse Gamma distribution with mean 0.01 and an infinite standard deviation.

3.3 Results

3.3.1 Data

The condition for Bayesian estimation is that the number of observables has to be less than or equal to the number of shocks in the model. Since there are two shocks in the model, the present empirical analysis is carried out using the St. Louis adjusted monetary base at quarterly frequencies for the period 1918:1 - 2014:2. Since the data is non-stationary and the since the model is log-linearized around the steady state, the series is HP-filtered prior to estimation.
3.3.2 Posterior Estimates

The right panel of Table 1 contains the summary statistics of the posterior distribution of the model parameters. The estimates include the mean, median, model, and 90% probability intervals of the model parameters. The posterior distribution of the parameters are plotted along with the prior distribution and the mode of the distribution in Figure 2. Most of the parameters are close to the prior mean. Of particular interest are the autoregressive parameters of the monetary base and of the nominal GDP target and the standard deviation of shocks to the monetary base and the nominal GDP target. First, the estimates suggest that both the monetary base and the nominal GDP target are more persistent than suggested by the prior means. Second, the standard deviation of shocks to the nominal GDP target are over 10 times larger than the standard deviation of shocks to the monetary base itself.

3.3.3 Impulse Response Functions

Figures 3 and 4 plot the impulse response functions of the variables in the model to shocks to the monetary base and the nominal GDP target, respectively. The impulse response functions are estimated using the mean of the posterior distribution. The shaded regions in Figures 3 and 4 represent the 90% probability intervals.

As shown in Figure 3, a shock to the monetary base results in an immediate increase in the monetary base, the monetary aggregate, nominal GDP, and real GDP. All of the variables subsequently exhibit monotonic decay. The responses of the variables are indicative of the monetary transmission mechanism. The increase in the monetary base increases nominal income, which increases deposits and therefore the monetary aggregate. The increase in the monetary aggregate is larger than the decline in the price of money and as a result, real money balances increase causing an increase in DM consumption and real GDP. The shock increases real GDP by 0.1% and persists for approximately 6 quarters after the shock.

Figure 4 plots the response of the variables to a shock to the nominal GDP target. A positive shock to the nominal GDP target results in a corresponding positive, hump-shaped response of the monetary base, the monetary aggregate, and real GDP. Again, the response is indicative of the monetary transmission mechanism. The increase in the monetary base from a shock to the nominal GDP target causes an increase in nominal GDP and a corresponding increase in deposits. The increase in the monetary aggregate is larger than the decline in the price of money, increasing real money balances and therefore
DM consumption and real GDP. In this case, however, the monetary base continues to increase after
the shock. The reason is due to the nature of the monetary policy rule. The posterior mean estimates
implies that the nominal income target and the monetary base are relatively persistent. As a result the
shock to the nominal GDP target implies that the monetary base peaks in the quarter after the shock.
The shock to the nominal GDP target increases real GDP by 0.5% at its peak and the effect persists for
approximately 15 - 16 quarters after the shock. The effect of the shock to the nominal GDP target is
therefore larger and more persistent than the shock to the monetary base itself. This lends support to
the view that changes in the monetary base that result from changes in the central bank’s nominal target
are more effective than than exogenous changes to the base itself.

4 Conclusion

In recent years, the Federal Reserve has quadrupled the monetary base. This unprecedented change
in policy is the result of a desire on the part of the Federal Reserve to increase real economic activity.
Some, such as Sumner (2011, 2012) and Woodford (2012) have suggested that this type of policy would
be more successful if it were tied to an explicit nominal income target. This paper presents a model
through which changes in the monetary base can have an effect on real economic activity and considers
two possible sources of changes in the monetary base. Specifically, the monetary base can change due
to a shock to the monetary base itself or the monetary base can change as a result of a change in the
central bank’s nominal target. Estimation of the model suggests that changes in the monetary base that
result from shocks to the central bank’s nominal target are larger and more persistent than shocks to the
monetary base itself. This paper therefore provides support for the view that changes in the monetary
base are more effective when they result from changes in the nominal target of the central bank.
References


Table 1: Prior and Posterior Distributions of the Parameters

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<th>Parameter</th>
<th>Prior Distribution</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Posterior Mean</th>
<th>Posterior Median</th>
<th>Posterior Mode</th>
<th>10% HPD</th>
<th>90% HPD</th>
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<td>0.988</td>
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<td>0.05</td>
<td>0.10</td>
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</table>
Figure 1: Prior Distributions of the Model Parameters
Figure 2: **Posterior Distribution of the Model Parameters.** The prior distribution of the parameters are plotted in gray. The posterior distribution is plotted in black. The vertical line denotes the mode of the distribution.
Figure 3: **Monetary Base Shock.** The thick black line plots impulse response functions of each of the variables evaluated at the posterior mean to a shock to the monetary base. The gray area corresponds to the 90% probability interval.
Figure 4: Nominal GDP Target Shock. The thick black line plots impulse response functions of each of the variables evaluated at the posterior mean to a shock to the nominal GDP target of the central bank. The gray area corresponds to the 90% probability interval.