Answers to 11.2

11.2 (a) Countries with high per capita income can decide whether to spend larger amounts on education than their poorer neighbours, or to spend more of their larger income on other things. They are likely to have more discretion with respect to where public monies are spent. On the other hand, countries with low per capita income may regard a particular level of education spending as essential, meaning that they have less scope for deviating from a mean function. These differences can be captured by a model with hetero-skedasticity.

(b) The least squares estimated function is

\[ \hat{y}_t = -0.1246 + 0.07317x_t \]

\[ R^2 = 0.862 \]

\[ (0.0485) \quad (0.00518) \]

This function and the corresponding residuals appear in Figure 11.1. The absolute magnitude of the errors does tend to increase as \( x \) increases suggesting the existence of heteroskedasticity.

![Figure 11.1 Estimated Function for Education Expenditure](image)

(c) Since it is suspected that, if heteroskedasticity exists, the variance is related to \( x_t \), we begin by ordering the observations according to the magnitude of \( x_t \). Then, splitting the sample into two equal subsets of 17 observations each, and applying least squares to each subset, we obtain \( \hat{\sigma}_1^2 = 0.0081608 \) and \( \hat{\sigma}_2^2 = 0.029127 \) leading to a Goldfeld-Quandt statistic of

\[ GQ = \frac{0.029127}{0.008161} = 3.569 \]

The critical value from an \( F \)-distribution with (15,15) degrees of freedom and a 5% significance level is \( F_{0.05} = 2.40 \). Since 3.569 > 2.40 we reject a null hypothesis of homoskedasticity and conclude that the error variance is directly related to per capita income \( x_t \).

(d) Using White's formula for standard errors our estimated regression line is

\[ \hat{y}_t = -0.1246 + 0.07317x_t \]

\[ (0.0392) \quad (0.00603) \]

The standard errors were found by taking square roots of the diagonal elements of Consistent Covariance of Estimates matrix produced by adding `/acov` option to the model statement in PROC REG in SAS.
Using $t_e = 2.037$ as the 5% critical value for 32 degrees of freedom, the two confidence intervals for $\beta_2$ are:

<table>
<thead>
<tr>
<th></th>
<th>lower</th>
<th>upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>least squares s.e.</td>
<td>0.0626</td>
<td>0.0837</td>
</tr>
<tr>
<td>White s.e.</td>
<td>0.0609</td>
<td>0.0854</td>
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The confidence interval that ignores the heteroskedasticity is narrower than the one that recognizes it, suggesting that ignoring heteroskedasticity will give us misplaced confidence about likely values of $\beta_2$.

(e) Generalized least squares estimation under the assumption $\text{var}(e_t) = \sigma^2 x_t$ yields

$$
\hat{y}_t = -0.0929 + 0.06932 x_t
$$

\begin{align*}
(0.0289) & \quad (0.00441)
\end{align*}

The estimated response of per capita education expenditure to per capita income has declined slightly relative to the least squares estimate. The associated 95% confidence interval is (0.0603, 0.0783). This interval is narrower than both those computed from least squares estimates. The comparison with the White-calculated interval suggests that generalized least squares is more efficient; a comparison with the conventional least squares interval is not really valid because the standard errors used to compute that interval are not valid.