

## Paired Sample Studies

- Paired Sample studies compare the same individuals before and after they have been subjected to certain treatment.
- The goal of such study is to determine whether the treatment caused a statistically significant change in response variable.

## Example: Does exercise help lose weight?

- Subjects: 5 volunteers
- Treatment: exercise
- Response variable: weight

• Data:	1	2	3	4	5
Before	168	180	220	150	170
After	160	173	209	151	170

## Example: Does exercise help lose weight?

- Null Hypothesis:  
Exercise makes no difference.  
Average weight loss = 0
- Alternative Hypothesis  
Exercise helps lose weight  
Average weight loss < 0

## Example: Does exercise help lose weight?

- This is a small sample → use  $t$ -statistic to do the significance test.
- Significance test:  $t_4 = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{-5 - 0}{5.24/\sqrt{5}} = -2.13$
- P-value: 0.049
- Conclusion:  
There is significant evidence against  $H_0$

## Example: Does exercise help lose weight?

- **What can be concluded from this study?**
  - We have statistically significant evidence to show that exercise helps lose weight.
  - We might have stronger evidence that exercise helps if we had a bigger sample.
  - Yes, we rejected  $H_0$ , but *how much* does the exercise really help?

## Paired Sample Study

- Preceding Weight Loss Example is a paired sample study.
- There is a pair of variables for each observation: “before” and “after”.
- Instead of using the actual data, we calculate the difference between “before” and “after” and test whether the average difference is significantly different from zero.

## Two-Sample Study

- Two-sample study compares two groups which are, typically, control and treatment group (e.g. compare the weight loss of people who were on a diet and those who weren't)
- If we find that those who dieted lost more weight, then we can say that weight loss was not due to *Placebo effect* and exercise really works.
- Two-Sample studies also can be used to compare two different groups – males vs. females, doctors vs. lawyers etc.

## Typical desing of the two-sample study:

- Null Hypothesis:  
The two groups are the same.
- Alternative Hypothesis  
The two groups are not the same.

## How to calculate test statistic for two samples

To compare two samples that don't have equal number of observations, we use this version of  $t$ -statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where  $\bar{x}_1$  and  $\bar{x}_2$  are the sample averages from the first and second sample and  $s_1^2$  and  $s_2^2$  are their variances.

## How to calculate test statistic for two samples

If the null hypothesis is that means our equal (like in our case) – we can simplify our test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu - \mu)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Appropriate degrees of freedom are from the smaller sample.

## Is There a Difference Between AM and PM classes?

- **Background Information:** Academic Performance may be different due to various factors:
  - more students with full time jobs in PM class;
  - students are tired;
  - two shorter meetings per week vs. one long meeting;
  - different academic backgrounds of students;
  - instructional quality is not the same.

## Is There a Difference Between AM and PM classes?

- **Study Design:**
  - Subjects are 92 students in Pr. L.'s class
    - 48 students in AM class
    - 44 students in PM class
  - Subject choose which class they want to take
  - Subjects are given 9 quizzes during the semester
  - Response variable: Quiz score
  - Explanatory variable: Section (AM or PM)

## Stating Hypotheses

- Null Hypothesis:

$$H_0: \mu_{PM} = \mu_{AM}$$

*Average quiz score in AM class is the same as the average quiz score in PM class.*

- Alternative Hypothesis

$$H_a: \mu_{PM} \neq \mu_{AM}$$

*Average quiz score in AM class is NOT the same as the average quiz score in PM class.*

## Is There a Difference Between AM and PM classes?

- Conclusion:

- Yes there is a difference!
- Test results may be invalid because distributions are not normal.
- Test DOES NOT prove that a student's scores would be higher if he/she were to enroll in the morning class.